WESTMINSTER BUSINESS SCHOOL

WORKING PAPER SERIES

2018/004

Time-varying SNP distribution and portfolio performance

Ángel León
Dept. Fundamentos del Análisis Económico,
Universidad de Alicante

Trino-Manuel Ñíguez
Westminster Business School,
University of Westminster

UNIVERSITY OF WESTMINSTER
VISION
STRATEGY
OPPORTUNITY

Time-varying semi-nonparametric distribution and portfolio performance

Ángel León

Dept. Fundamentos del Análisis Económico, Universidad de Alicante,

Campus San Vicente del Raspeig, Alicante 03080, Spain.

Email: aleon@ua.es

Trino-Manuel Ñíguez*

Westminster Business School, University of Westminster,

35 Marylebone Road, London NW1 5LS, UK.

Email: T.M.Niguez@wmin.ac.uk

August 31, 2018

Abstract

First, we propose a novel method in the operations research of portfolio selection based on equity screening rules determined by performance measures (PMs) under a framework of time-varying (TV) semi-nonparametric (SNP) densities for asset returns with conditional volatility such as the popular GJR specification. The TV-SNP density, as well as providing an improved fit of the return tail dynamics due to the flexibility driven by the TV higher-order moments, allows for tractable closed-form expressions of both conditional reward and risk measures, specifically partial moments and expected shortfall. Second, as a result, we show that some conditional PMs such as Rachev and skewness-kurtosis ratios yield portfolios that dominate, in our out-of-sample (OOS) empirical study for stocks from the S&P 100 index, those under the conditional Sharpe, Sortino and Omega ratios. Third, we highlight as cutting-edge the use of conditional copula models to provide more evidence on the tail dependence of the OOS portfolio return distributions under different PMs with respect to the benchmark Sharpe ratio portfolio. Finally, a strict robustness analysis hinged on alternative portfolio rebalancing frequencies and weighting schemes is also carried out.

Keywords: Investment analysis; conditional higher-order moments; copula; equity screening; expected shortfall.

*Corresponding author

1 Introduction

Optimal capital allocation relies critically on the modeling of asymmetry and tail-fatness of portfolio return distributions. Recent econometric results have shown the importance of clustering as well as asymmetric response of conditional high-order moments to positive and negative shocks; see Jondeau and Rockinger (2003) (JR hereafter) for evidence shown under the skewed t distribution; León, Rubio and Serna (2005) for the case of Gram-Charlier distributions; Feunou, Jahan-Parvar and Tédongap (2016) for a model with skewed GED dynamic higher-order moments; and Theodossiou and Savva (2016) for the time-varying skewed generalized t (SGT) distribution.[1]

Following this literature, we propose a probability density function (pdf) that features practicability, flexibility and accuracy for financial risk management, asset allocation and asset pricing. This pdf for modeling standardized asset returns extends the semi-nonparametric (SNP) density of León, Mencía and Sentana (2009) through both time-varying (TV) skewness and kurtosis. Hereafter, we refer to this distribution as TV-SNP. Our paper proceeds as follows. First, we study the parametric properties and obtain the expression of the log-likelihood function of the TV-SNP distribution with a GARCH family specification, in particular, the Glosten, Jagannathan and Runkle (1993) model. Henceforth, this augmented model is named as TV-SNP-GJR. Then, we analyze this model estimation results over a set of daily stock-index and foreign-exchange returns. Furthermore, we implement the impact curve (NIC) analysis in Anatolyev and Petukhov (2016) to the conditional skewness and kurtosis of our model to study their asymmetric responsiveness to shocks.

Second, we use our parametric model for portfolio management. We precisely construct active portfolio strategies through equity screening rules, as a possible alternative to the large-scale portfolio optimization problem,[2] based on time-varying performance measures (PMs) used to rank stocks and then, building portfolios with the highest-ranking ones. These PMs extend those in previous studies which are defined under an unconditional framework, and implicitly gather the dynamics of the return higher-order moments yielding more precise reward-to-risk ratios. The conditional PMs we use are the following: (a) The Sharpe ratio (SR) (Sharpe, 1966, 1994) as the benchmark. (b) The skewness and kurtosis ratio (SKR), see Watanabe (2006). (c) PMs based on partial moments, such as (i) the Farinelli-Tibiletti (FT) ratio, which nests the popular Omega and Upside potential ratios, see Farinelli and Tibiletti (2008), and (ii) the Sortino ratio, see Sortino and Van der Meer (1991). (d) Quantile-based PMs, such as the Rachev or expected tail ratio (ETR), and the Value-at-Risk ratio (VaRR). See Biglova, Ortobelli, Rachev and Stoyanov (2004) and Caporin and Lisi (2011) for these two last measures, respectively.[3]

Third, we analyze the out-of-sample performance of portfolios composed from selecting among the stocks that constitute the S&P 100 index. These stocks are evaluated under each PM strategy at alternative rebalancing periods to obtain rankings used to form portfolios. We start by considering equally-weighted portfolios and then study other portfolios driven by alternative weighting schemes, such as the shortsale-constrained global-minimum-variance, the volatility timing and the reward-to-risk timing; see Kirby and Ostdiek (2012). We employ conditional copula models to focus on the tail dependence between the return distributions obtained under the SR and each alternative PM strategy; see Patton (2006, 2013). Finally, our results under the TV-SNP-GJR are reinforced through a comparative analysis respecting the approach of

[1] The SGT was firstly introduced by Theodossiou (1998).

[2] A feasible solution to shrinking the dimension of the large portfolio optimization problem consists on applying the principal component analysis, which has been implemented for the case of fixed income portfolios in Ortobelli, Vitali, Cassader and Tichy (2018).

[3] In order to address the unreliability in portfolio rankings when PMs, such as SR, SKR and Sortino, yield negative values, we use versions of these PMs modified according to the methodology in Israelsen (2005).

historical simulation.

Our empirical results show evidence on the relevance of skewness and kurtosis dynamics since the TV-SNP-GJR models improve the goodness-of-fit of the constant skewness and kurtosis SNP (C-SNP-GJR) model. Furthermore, not all portfolios selected according to the time-varying PM strategies yield greater cumulative returns regarding those obtained under the benchmark SR. Namely, the portfolios based on partial moments (Sortino, Omega and Upside potential) differ less to the SR than the rest of the PMs. We find considerable gains in both SKR and ETR portfolio cumulative returns, indeed. As a final remark, our results reconcile the conflicting evidence reported in the literature on the importance of higher-order moments (departure of return distributions from normality) for ranking portfolios.[4]

The remainder of the article is organized as follows. In Section 2, we define the TV-SNP-GJR model, discuss its statistical properties and obtain closed-form expressions for the expected shortfall and partial moments used to build conditional parametric PMs. Section 3 discusses the model estimation through an empirical application to stock index and FX returns. Section 4 presents the conditional PMs used in our analysis. Section 5 shows the performance of out-of-sample portfolios through equity screening based on PMs for ranking stocks that compose the S&P 100 index. In Section 6, we summarize our conclusions. All of the proofs are provided in the Appendix.

2 Modeling stock returns

Let the stock return r_t be a process characterized by the sequence of conditional densities $f\left(r_t\left|I_{t-1};\psi\right.\right)$, where I_{t-1} denotes the information set available at $t-1$ containing past values of r_t, $\psi=\left(\theta',v'\right)'$ is the vector of unknown parameters such that θ is the subset characterizing both the conditional mean and variance of r_t, i.e. $\mu_t\left(\theta\right)=\mu\left(I_{t-1};\theta\right)$ and $\sigma_t^2\left(\theta\right)=\sigma\left(I_{t-1};\theta\right)$, and finally, v is the subset characterizing the shape of the distribution of the standardized observations, z_t. Thus, we assume that

$$r_t=\mu_t\left(\theta\right)+\varepsilon_t,\qquad \varepsilon_t=\sigma_t\left(\theta\right)z_t,\qquad z_t\sim iid\ g\left(z_t;v\right). \tag{1}$$

So, equation (1) decomposes the return at time t into a conditional mean, μ_t, and a gross innovation, ε_t. The term ε_t is defined as the product between the conditional volatility, σ_t, and the standardized innovation, z_t. It is assumed that $\{z_t\}$ is a sequence of independent identically distributed random variables with $g\left(\cdot\right)$ as pdf.

2.1 SNP density of z_t

Let us define z_t as a linear transformation of x_t with pdf given by the SNP class of distributions introduced by Gallant and Nychka (1987), and León et al. (2009) who studied its parametric properties. Specifically,

$$z_t=a\left(v\right)+b\left(v\right)x_t,\qquad b=1/\sigma_x,\quad a=-b\mu_x, \tag{2}$$

[4] On the one hand, there is a stream of research, that includes the papers by Eling and Schuhmacher (2007), Eling (2008) and Auer (2015), that advocates the choice of PM is irrelevant for portfolio evaluation. On the other hand, some studies, such as Zakamouline (2011) and León and Moreno (2017), showed that higher-order moments of distribution do play a crucial role for portfolio evaluation. Finally, the empirical evidence in León, Navarro and Nieto (2018), who also implement a screening rule strategy but using sample estimations of PMs instead of a parametric approach, reinforces our results about the best performance of portfolios under the Generalized Rachev ratio, which nests the ETR used in our study.

where $\mu_x = E(x_t)$ and $\sigma_x = \sqrt{V(x_t)}$ are, respectively, the mean and the standard deviation of x_t with density function

$$q_n(x_t) = \frac{\phi(x_t)}{v'v}\left(\sum_{k=0}^{n} v_k H_k(x_t)\right)^2, \tag{3}$$

where $v = (v_0, v_1, \ldots, v_n)' \in \mathbb{R}^{n+1}$, $\phi(\cdot)$ denotes the pdf of a standard normal random variable and $H_k(\cdot)$ are the normalized Hermite polynomials. These polynomials can be defined recursively for $k \geq 2$ as

$$H_k(x) = \frac{xH_{k-1}(x) - \sqrt{k-1}H_{k-2}(x)}{\sqrt{k}}, \tag{4}$$

with initial conditions $H_0(x) = 1$ and $H_1(x) = x$. The set $\{H_k(x)\}_{k\in N}$ constitutes an orthonormal basis with respect to the weighting function $\phi(x)$. Thus, $E_\phi[H_k(x)H_l(x)] = \mathbf{1}_{(k=l)}$, where $\mathbf{1}_{(\cdot)}$ is the usual indicator function and the operator $E_\phi[\cdot]$ takes the expectation of its argument with respect to $\phi(\cdot)$ as pdf.

Since $q_n(\cdot)$ in (3) is homogeneous of degree zero in v, we impose $v_0 = 1$ to solve the scale indeterminacy. If we consider $n = 2$ and expand the square term expression in (3), we obtain an alternative expression of $q_2(\cdot)$ and, henceforth, denoted as $q(\cdot)$:

$$q(x_t) = \phi(x_t)\sum_{k=0}^{4}\gamma_k(v)H_k(x_t), \tag{5}$$

such that

$$\begin{aligned}
\gamma_0(v) &= 1, \quad \gamma_1(v) = \frac{2v_1\left(1+\sqrt{2}v_2\right)}{v'v}, \\
\gamma_2(v) &= \frac{\sqrt{2}\left(v_1^2 + 2v_2^2 + \sqrt{2}v_2\right)}{v'v}, \qquad (6)\\
\gamma_3(v) &= \frac{2\sqrt{3}v_1v_2}{v'v}, \quad \gamma_4(v) = \frac{\sqrt{6}v_2^2}{v'v}. \qquad (7)
\end{aligned}$$

2.1.1 Moments

The first four noncentral moments of x_t with pdf in (5) are:

$$\begin{aligned}
\mu'_x(1) &= \gamma_1(v), \\
\mu'_x(2) &= \sqrt{2}\gamma_2(v) + 1, \\
\mu'_x(3) &= \frac{6v_1\left(1+2\sqrt{2}v_2\right)}{v'v}, \\
\mu'_x(4) &= \frac{12\left(v_1^2 + 3v_2^2 + \sqrt{2}v_2\right)}{v'v} + 3. \qquad (8)
\end{aligned}$$

Hence, $\mu_x = \mu'_x(1)$ and $\sigma_x^2 = \mu'_x(2) - \mu_x^2$. Therefore, the skewness and kurtosis of z_t are given by

$$s_z \equiv E\left(z_t^3\right) = a^3 + 3a^2b\mu'_x(1) + 3ab^2\mu'_x(2) + b^3\mu'_x(3), \tag{9}$$

$$k_z \equiv E\left(z_t^4\right) = a^4 + 4a^3b\mu'_x(1) + 6a^2b^2\mu'_x(2) + 4ab^3\mu'_x(3) + b^4\mu'_x(4). \tag{10}$$

2.1.2 Cumulative distribution function (cdf)

Let $Q(\cdot)$ denote the cdf related to x_t with $q(\cdot)$ as pdf defined in (5). The pdf of z_t is given by $g(z_t) = \frac{1}{b(v)}q\left(\frac{z_t - a(v)}{b(v)}\right)$. The next result shows the expression of the cdf related to z_t.

Proposition 1. *The cdf of z_t in (2), denoted as $G(\cdot)$, is obtained as*

$$\begin{aligned} G(z_t) &= Q\left(z_t^*\right) = \int_{-\infty}^{z_t^*} q\left(x_t\right) dx_t \\ &= \Phi\left(z_t^*\right) - \phi\left(z_t^*\right) \sum_{k=1}^{4} \frac{\gamma_k}{\sqrt{k}} H_{k-1}\left(z_t^*\right), \end{aligned} \tag{11}$$

where $z_t^* = \left(z_t - a\right)/b$, $H_k(\cdot)$ is given in (4) and $\Phi(\cdot)$ denotes the cdf of the standard normal random variable.

Proof. *It is verified that $\int_{-\infty}^{u} H_k(x)\phi(x)dx = -\frac{1}{\sqrt{k}} H_{k-1}(u)\phi(u)$, then (11) is directly obtained.*

2.2 GJR-SNP model and moments of ε_t

Let $\sigma_t^2 = E\left[\varepsilon_t^2 \left| I_{t-1}\right.\right]$ be the conditional variance model suggested by Glosten et al. (1993), GJR hereafter. Then,

$$\begin{aligned} \sigma_t^2 &= \alpha_0 + \beta\sigma_{t-1}^2 + \alpha_1^+ \left(\varepsilon_{t-1}^+\right)^2 + \alpha_1^- \left(\varepsilon_{t-1}^-\right)^2 \\ &= \alpha_0 + \beta\sigma_{t-1}^2 + \alpha_1^+ \sigma_{t-1}^2 \left(z_{t-1}^+\right)^2 + \alpha_1^- \sigma_{t-1}^2 \left(z_{t-1}^-\right)^2, \end{aligned} \tag{12}$$

such that $\alpha_0 > 0$, $\beta \geq 0$, $\alpha_1^+ \geq 0$ and $\alpha_1^- \geq 0$. We use the notation $y_t^+ = \max\left(y_t, 0\right)$, $y_t^- = \min\left(y_t, 0\right)$ where y_t can be either ε_t or z_t defined in (1). Another representation of (12) is given by $\sigma_t^2 = \alpha_0 + \beta\sigma_{t-1}^2 + \left(\alpha_1 + \gamma D_{t-1}\right)\varepsilon_{t-1}^2$ such that $D_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ and $D_{t-1} = 0$ if $\varepsilon_{t-1} \geq 0$. Hence, both expressions are related through $\alpha_1^+ = \alpha_1$ and $\alpha_1^- = \alpha_1 + \gamma$. Henceforth, we denote (12) as the GJR (1,1) model (or simply GJR) which nests the GARCH (1,1) model when $\alpha_1^+ = \alpha_1^-$. A similar specification to (12) for the conditional volatility σ_t, instead of σ_t^2, is suggested by Zakoïan (1994).

If we assume (12) to be covariance stationary, then the unconditional variance of ε_t is obtained as

$$\sigma_\varepsilon^2 \equiv E\left(\sigma_t^2\right) = \frac{\alpha_0}{1-\omega_1}, \qquad 0 < \omega_1 < 1, \tag{13}$$

where

$$\omega_1 = \beta + \alpha_1^+ + \left(\alpha_1^- - \alpha_1^+\right) E\left[\left(z_t^-\right)^2\right]. \tag{14}$$

Following He and Terasvirta (1999), it can be shown that the unconditional kurtosis is given by[5]

$$k_\varepsilon = \frac{E\left(\varepsilon_t^4\right)}{\sigma_\varepsilon^4} = k_z \frac{E\left(\sigma_t^4\right)}{\sigma_\varepsilon^4} = k_z A\left(\theta, v\right), \tag{15}$$

where k_z is defined in (10) and

$$A\left(\theta, v\right) = \frac{1-\omega_1^2}{1-\omega_2} \geq 1, \tag{16}$$

such that ω_2 is the condition for the existence of the fourth-order moment,

$$\omega_2 = 2\beta\omega_1 - \beta^2 + \left(\alpha_1^+\right)^2 k_z + \left[\left(\alpha_1^-\right)^2 - \left(\alpha_1^+\right)^2\right] E\left[\left(z_t^-\right)^4\right] < 1. \tag{17}$$

Respecting the unconditional skewness of ε_t, that is obtained as $s_\varepsilon = E\left(\varepsilon_t^3\right)/\sigma_\varepsilon^3 = s_z E\left(\sigma_t^3\right)/\sigma_\varepsilon^3$ with s_z in (9), we cannot obtain a closed-form expression. Nonetheless, if we assume $\mu_t = \mu$, then s_ε and k_ε would be the unconditional skewness and kurtosis of the stock returns.

[5] We can rewrite (12) as $\sigma_t^2 = \alpha_0 + c_t\sigma_{t-1}^2$ where $c_t = \beta + \alpha_1^+ \left(z_{t-1}^+\right)^2 + \alpha_1^- \left(z_{t-1}^-\right)^2$. Then, $\sigma_t^4 = \alpha_0^2 + c_t^2\sigma_{t-1}^4 + 2\alpha_0 c_t \sigma_{t-1}^2$. By taking expectations and assuming that $E\left(\sigma_t^4\right) = E\left(\sigma_{t-1}^4\right)$, then we obtain $\left[1 - E\left(c_t^2\right)\right] E\left(\sigma_t^4\right) = \alpha_0^2 + 2\alpha_0 E\left(c_t\right) E\left(\sigma_t^2\right)$. If we denote $E\left(c_t\right)$ and $E\left(c_t^2\right)$, respectively, as ω_1 and ω_2, we finally obtain (15).

Finally, the expressions $E\left[\left(z_t^-\right)^2\right]$ and $E\left[\left(z_t^-\right)^4\right]$, defined in (14) and (17) respectively, can be obtained according to the following result:

Proposition 2. *Let $z_t = a + bx_t$ be the standardized variable defined in* (2) *and x_t an iid sequence with pdf given in* (5)*, then*

$$\begin{aligned} E\left[\left(z_t^-\right)^k\right] &= \textstyle\int_{-\infty}^{0} z_t^k g\left(z_t\right) dz_t = \int_{-\infty}^{-a/b} \left(a + bx_t\right)^k q\left(x_t\right) dx_t \\ &= \sum_{j=0}^{k} \binom{k}{j} a^{k-j} b^j \xi_j\left(-a/b\right), \end{aligned} \tag{18}$$

where $k \in \mathbb{N}$ and $\xi_j\left(u\right) = \int_{-\infty}^{u} x^j q\left(x\right) dx$ is given by (48) *obtained in section ii) of the Appendix.*

Note that $z_t \sim iid\ N\left(0,1\right)$ when $v_1 = v_2 = 0$ under the SNP distribution for x_t, then $s_z = 0$ and $s_\varepsilon = 0$. It can also be shown that $E\left[\left(z_t^-\right)^2\right] = 1/2$, $E\left[\left(z_t^-\right)^4\right] = 3/2$, $k_z = 3$ and so, k_ε in (15) becomes the following well-known expression:

$$k_\varepsilon = 3\left(\frac{1 - \beta^2 - \beta\left(\alpha_1^+ + \alpha_1^-\right) - \frac{1}{4}\left(\alpha_1^+ + \alpha_1^-\right)^2}{1 - \beta^2 - \beta\left(\alpha_1^+ + \alpha_1^-\right) - \frac{3}{2}\left[\left(\alpha_1^+\right)^2 + \left(\alpha_1^-\right)^2\right]}\right). \tag{19}$$

2.3 Time-varying SNP parameters

Let ε_t be the gross innovation in (1) and let $\sigma_t^2 = E\left[\varepsilon_t^2 \left| I_{t-1}\right.\right]$ follow the GJR model in (12). Then, the conditional skewness and kurtosis of r_t are defined, respectively, as

$$s_{r,t} = \frac{E\left(\varepsilon_t^3 \left| I_{t-1}\right.\right)}{\sigma_t^3}, \qquad k_{r,t} = \frac{E\left(\varepsilon_t^4 \left| I_{t-1}\right.\right)}{\sigma_t^4}. \tag{20}$$

If we let the SNP distribution exhibit time-varying parameters, the pdf of x_t in (5) is now defined as $q\left(x_t \left| I_{t-1}\right.\right)$ where v_i is replaced with $v_{i,t}$ $(i = 1, 2)$ being measurable with respect to the information set I_{t-1}. Hence, $s_{r,t} = s_{z,t}$ and $k_{r,t} = k_{z,t}$ are now time-varying such that both $s_{z,t}$ and $k_{z,t}$ are obtained by plugging $v_{i,t}$ into equations (9) and (10), respectively. This way of modeling the conditional higher moments through a rather complex non-linear mapping is the most popular one in the literature and it is known as the *indirect approach*. We model $v_{i,t}$ according to the autoregressive framework by Anatolyev and Petukhov (2016) (AP hereafter):

$$v_{i,t} = \varphi_{0i} + \varphi_{1i} v_{i,t-1} + \Upsilon_i\left(z_{i,t-1}\right), \tag{21}$$

where $\Upsilon_i\left(\cdot\right)$ is a real-valued function that aims to capture the *news impact curve* (NIC) specification of both conditional skewness and kurtosis.[6] In this paper we will mainly use the parametric "asymmetric" specification borrowed from JR:

$$\Upsilon_i\left(z\right) = \varphi_{2i}^+ z^+ + \varphi_{2i}^- z^-, \tag{22}$$

such that $z^+ = \max\left(z, 0\right)$, $z^- = \min\left(z, 0\right)$ and $\varphi_{2i}^+ \neq \varphi_{2i}^-$. Note that (22) shows an asymmetric linear specification that can also be found, among others, in Feunou et al. (2016) and Lalancette and Simonato (2017). Another example of NIC, $\Upsilon_i\left(z\right)$, that will be used is the "transition" specification which is defined as

$$\Upsilon_i\left(z\right) = \varphi_{2i}(1 + \varphi_{3i}\left|z\right|)z. \tag{23}$$

[6] According to the literature, two choices are suggested for the equations driven by the TV-SNP parameters, v_{it}. The first is as a function on lags of the standardized returns z_t, and the second as a function on lags of ε_t . We stick to the first choice since we are indeed modeling the higher-moments for the distribution of the standardized returns.

We consider mainly two particular cases for $v_{i,t}$ specifying or not the autoregressive (AR) component in (21):

$$v_{i,t} = \varphi_{0i} + \varphi_{2i}^{+} z_{i,t-1}^{+} + \varphi_{2i}^{-} z_{i,t-1}^{-}, \tag{24}$$

$$v_{i,t} = \varphi_{0i} + \varphi_{1i} v_{i,t-1} + \varphi_{2i}^{+} z_{i,t-1}^{+} + \varphi_{2i}^{-} z_{i,t-1}^{-}. \tag{25}$$

2.4 Log-likelihood function

Note that we have previously studied the main components that define the stock return equation given in (1). If we now express the conditional density of r_t in terms of the conditional density of x_t, then

$$f\left(r_t \left| I_{t-1}; \psi \right.\right) = \frac{q\left(x_t \left| I_{t-1} \right.\right)}{b\left(v_t\right) \sigma_t}, \tag{26}$$

where ψ is the parameter set, $q\left(\cdot \left| I_{t-1} \right.\right)$ is the conditional pdf given in (5) but with time-varying parameters $v_{1,t}$ and $v_{2,t}$, $x_t = \frac{z_t(\theta) - a(v_t)}{b(v_t)}$ and $z_t\left(\theta\right) = \left(r_t - \mu_t\left(\theta\right)\right)/\sigma_t\left(\theta\right)$. The log-likelihood function corresponding to a particular observation r_t, denoted as l_t, takes the following form:

$$\begin{aligned} l_t &= -\frac{1}{2}\ln\left(\sigma_t^2\left(\theta\right)\right) - \ln\left(b\left(v_t\right)\right) - \ln\left(v_t' v_t\right) - \frac{1}{2}\ln\left(2\pi\right) \\ &\quad -\frac{1}{2}\left(\frac{z_t\left(\theta\right) - a\left(v_t\right)}{b\left(v_t\right)}\right)^2 + \ln\left[\sum_{k=0}^{2} v_{k,t} H_k\left(\frac{z_t\left(\theta\right) - a\left(v_t\right)}{b\left(v_t\right)}\right)\right]^2, \end{aligned} \tag{27}$$

such that $v_{0,t} = 1$, $v_{i,t} = v_{i,t}\left(\vartheta_i\right)$ where $\vartheta_i \subset \psi$ is the parameter set underlying the equation of $v_{i,t}$ in (21).

2.5 Conditional quantile and expected shortfall

Let $F\left(r_t \left| I_{t-1} \right.\right)$ denote the conditional cumulative distribution function (cdf) corresponding to the *TV-SNP* model of r_t with pdf in (26),

$$F\left(r_t \left| I_{t-1} \right.\right) = \int_{-\infty}^{r_t} f\left(r_t \left| I_{t-1}; \psi \right.\right) dr_t = \int_{-\infty}^{r_t^*} q\left(x_t \left| I_{t-1} \right.\right) dx_t = Q\left(r_t^* \left| I_{t-1} \right.\right), \tag{28}$$

where $Q\left(\cdot \left| I_{t-1} \right.\right)$ is the conditional cdf, which is just the cdf $Q\left(\cdot\right)$ in (11) but with TV-SNP parameters, and $r_t^* = \left(r_t - \mu_t - a_t\sigma_t\right)/b_t\sigma_t$ where $a_t \equiv a\left(v_t\right)$ and $b_t \equiv b\left(v_t\right)$. The α-quantile, or VaR at the α confidence level, of the distribution of the stock return r_t is $r_{\alpha,t} \equiv F^{-1}(\alpha \left| I_{t-1} \right.)$.[7] So,

$$r_{\alpha,t} = \kappa_{0t} + \kappa_{1t} Q_t^{-1}\left(\alpha\right), \tag{29}$$

where $\kappa_{0t} = \mu_t + a_t\sigma_t$, $\kappa_{1t} = b_t\sigma_t$ and $Q_t^{-1}\left(\alpha\right) \equiv \inf\left\{x \left| Q(x \left| I_{t-1} \right.) \geq \alpha \right.\right\}$ is the conditional α-quantile with $q\left(\cdot \left| I_{t-1} \right.\right)$ as pdf. Since $q\left(\cdot \left| I_{t-1} \right.\right)$ nests the $N\left(0,1\right)$ distribution for $v_{1,t} = v_{2,t} = 0$, then $Q_t^{-1}\left(\alpha\right) = \Phi^{-1}\left(\alpha\right) = z_\alpha$. Once we have obtained $r_{\alpha,t}$ in (29), the expected shortfall (ES) is easily computed.

Proposition 3. *Let r_t be the stock return with pdf in* (26) *and let $r_{\alpha,t}$ be the conditional α-quantile in* (29)*, then*

$$\begin{aligned} ES_t\left(\alpha\right) &= E_{t-1}\left(r_t \left| r_t \leq r_{\alpha,t} \right.\right) \\ &= \kappa_{0t} + \kappa_{1,t} E_{t-1}\left(x_t \left| x_t \leq r_{\alpha,t}^* \right.\right) \\ &= \kappa_{0t} + \frac{\kappa_{1t}}{\alpha}\xi_{1t}\left(r_{\alpha,t}^*\right) \end{aligned} \tag{30}$$

where $r_{\alpha,t}^* = \left(r_{\alpha,t} - \kappa_{0t}\right)/\kappa_{1t}$ and $\xi_{1t}\left(u\right) = \int_{-\infty}^{u} xq\left(x \left| I_{t-1} \right.\right) dx$ is the conditional version of $\xi_1\left(u\right) =$ $\int_{-\infty}^{u} xq\left(x\right) dx$ that *is obtained in section ii) in the Appendix.*

Proof. *See section iii) in the Appendix.*

[7] Note that $r_{\alpha,t} \equiv F^{-1}(\alpha \left| I_{t-1} \right.)$ can be obtained by using, for instance, the Matlab function named *fzero* (finding roots of nonlinear equations).

2.6 Conditional partial moments

The lower partial moments (LPMs), see Fishburn (1977), measure risk by negative deviations of the stock return in relation to a return threshold, θ. The conditional LPM of order m where the stock return r_t follows a TV-SNP process, i.e. with pdf given by (26), is defined as

$$LPM_t(\theta, m) = \int_{-\infty}^{\theta} (\theta - r_t)^m f(r_t \,|I_{t-1}) dr_t. \tag{31}$$

The LPMs are asymmetric risk measures in contrast to the popular symmetric risk (standard deviation).

The conditional upper partial moment (UPM) of order m and return threshold θ is defined as

$$UPM_t(\theta, m) = \int_{\theta}^{\infty} (r_t - \theta)^m f(r_t \,|I_{t-1}) dr_t. \tag{32}$$

In this paper we are only interested in the LPMs of orders 1 and 2, that is, $LPM_t(\theta, 1)$ and $LPM_t(\theta, 2)$ in (31). Respecting the UPMs, we use $UPM_t(\theta, 1)$ in (32). Next, we obtain the closed-form expressions of the two LPMs and the UPM.

Proposition 4. *Let r_t be the stock return driven by the TV-SNP process with pdf in* (26)*, then*

$$\begin{aligned} LPM_t(\theta,1) &= (\theta - \kappa_{0t})\,\xi_{0t}\,(\theta_t^*) - \kappa_{1t}\xi_{1t}\,(\theta_t^*)\,, && (33)\\ LPM_t(\theta,2) &= (\theta - \kappa_{0t})^2\,\xi_{0t}\,(\theta_t^*) + \left(\kappa_{1t}^2 - 2\theta\kappa_{1t}\right)\xi_{1t}\,(\theta_t^*) + \kappa_{1t}^2\xi_{2t}\,(\theta_t^*)\,, && (34)\\ UPM_t(\theta,1) &= \mu_t - \theta + LPM_t(\theta,1), && (35) \end{aligned}$$

where $\kappa_{0t} = \mu_t + a_t\sigma_t$, $\kappa_{1t} = b_t\sigma_t$, $\theta_t^* = (\theta - \kappa_{0t})/\kappa_{1t}$, $\xi_{0t}(u) = Q(u\,|I_{t-1})$ and $\xi_{jt}(u) = \int_{-\infty}^{u} x^j q(x\,|I_{t-1})\,dx$ is the conditional version of $\xi_j(u) = \int_{-\infty}^{u} xq(x)\,dx$ obtained in section ii) of the Appendix.

Proof. *It is obtained straightforwardly.*

3 Estimation

3.1 Dataset and summary statistics

We start analyzing the time-series behavior of six stock indexes and four foreign exchange (FX) rates. The data employed were (daily) percentage log returns, which were computed as $r_t = 100\log(P_t/P_{t-1})$ from series $\{P_t\}_{t=1}^T$ of daily closing prices for Nasdaq, TAIEX, Bovespa, CAC, DAX and EUROSTOXX stock indexes; and pound sterling to euro (UK-EU), Japanese yen to U.S. dollar (JAP-US), Canadian dollar to U.S. dollar (CAN-US) and pound sterling to U.S. dollar (UK-US) FX rates. All of the price series were sampled from September 28, 1997 to September 27, 2017 to obtain a total of $T = 5,219$ observations. The data were obtained from Datastream.

Table 1 exhibits summary statistics of both stock-index and FX returns. The means, medians, standard deviations and ranges (minima and maxima values) are comparable to those observed in other studies. Clearly, all the series show high leptokurtosis with the UK-US returns presenting the largest kurtosis (14.7), and the TAIEX the smallest (6.79). The degree of unconditional skewness is heterogeneous among the series, with the largest negative (in absolute value) and positive skewness corresponding to the JAP-US (-0.47) and UK-US (0.57) returns, respectively, and the smallest to the Nasdaq (-0.06). The UK-EU and UK-US returns are positively skewed whilst the rest of the series present negative skewness. FX returns tend to exhibit larger skewness than stock-index returns. In all cases, the Jarque-Bera (J-B) test rejects the null of normality, motivating the use of our SNP distribution.

Table 1: Summary statistics for daily percent stock-index and foreign-exchange log returns

	Nasdaq	TAIEX	Bovespa	CAC	DAX	EUROSTOXX	UK-EU	JAP-US	CAN-US	UK-US
Mean	0.0258	0.0022	0.0151	0.0120	0.0228	0.0074	0.0045	-0.0014	-0.0022	0.0035
Median	0.0550	0.0000	0.0568	0.0453	0.0542	0.0247	-0.0060	0.0047	0.0000	0.0000
Std. dev.	1.5894	1.4922	2.4685	1.5738	1.6170	1.5942	0.5115	0.6837	0.5608	0.5791
Min	-10.1684	-11.3418	-17.9631	-11.7370	-9.6010	-11.1024	-2.6972	-6.5818	-5.0462	-4.4737
Max	13.2546	8.2571	18.0149	12.1434	12.3697	11.9653	6.2208	3.7102	4.3375	8.3113
Skewness	-0.0559	-0.2196	-0.2431	-0.0578	-0.1088	-0.0844	0.5047	-0.4724	-0.1013	0.5739
Kurtosis	8.5901	6.7904	9.3518	9.0182	7.5353	8.3929	9.0644	8.0969	8.4326	14.7428
J-B stat	6796.9	3165.6	8823.2	7877.6	4482.2	6329.5	8217.5	5842.2	6425.6	30226

This table presents the summary statistics for stock-index and FX daily percent log returns from September 29, 1997 to September 27, 2017 (5,218 obs.). The Jarque-Bera (J-B) statistic is asymptotically distributed as a Chi-square with two degrees of freedom, χ^2_2. The critical value of χ^2_2 for the 5% significance level is 5.99.

3.2 Estimation results

The parameters of the SNP models we considered in this analysis were estimated using maximum likelihood (ML) according to equation (27). To account for the small structure in the return conditional means, we filtered the r_t series with autoregressive processes of different orders for the conditional mean, μ_t. Since the estimations, under either filtered ($r_t - \widehat{\mu}_t$) or non-filtered returns, yielded rather similar results, we decided to report only the results for non-filtered data. Therefore, we adopt a constant mean equation for r_t, i.e. $\mu_t = \mu$. The stylized features of returns volatility were described through the GJR process in (12).

To model skewness and kurtosis, we started with a constant SNP (C-SNP) specification, where $v_{i,t}$, $i = 1, 2$ are constant, i.e. $(v_{1,t}, v_{2,t}) = (\varphi_{01}, \varphi_{02})$. This structure is nested in equations (24) and (25) which allow for time-varying $v_{i,t}$. Note that, unlike Hansen's (1994) skewed Student-t, there are no constraints on the C-SNP parameters in order to ensure a well-defined pdf. In short, the C-SNP becomes simpler and so, easier to estimate.

Next, we consider the TV1-SNP model, defined in (24), which specifies $v_{i,t}$ directly as function of past positive and negative standardized residuals, z_t^+ and z_t^-.[8] The TV1-SNP model provides the C-SNP with flexibility to capture dynamics in skewness and kurtosis such as clustering and asymmetric responses to positive and negative shocks.

Finally, we implement the TV2-SNP model, defined in (25), which includes AR(1) structure in $v_{i,t}$. Note that (25) resembles the GJR process for the conditional variance, although its parameters do not have the same interpretation as in the GJR. To avoid local optima, starting parameter values were obtained by using the adaptive simulated annealing optimization algorithm applied sequentially to nested specifications beginning from the Normal-GJR model.

In Tables 2-4 , we report the results of the various estimations. Table 2 presents the estimation results of model C-SNP-GJR. The parameter estimates of both mean and variance equations are in line with those reported in numerous studies. The unconditional mean parameter, μ, is not significant for any of the return series, except for the DAX returns for which it is significant at the five per cent level. The parameter estimates of the variance equation show that, for all series, the model correctly captures the asset returns stylized features of (i) clustering and high persistence in volatility, and (ii) asymmetric response of volatility to positive and negative shocks. The condition for the existence of the unconditional second moment of ε_t, given by ω_1 in (14), is satisfied for all series. In all cases, the unconditional standard deviations implied by model C-SNP-GJR are very close to the sample ones. For instance, the estimated σ_ε in (13) equals 2.46 and the sample standard deviation is 2.47 for Bovespa. Both persistence, β, and asymmetry, $\alpha_1^- \neq \alpha_1^+$, at the level of volatility in (12) are not altered either through the Normal or the different SNP specifications. Similar results have been reported by JR for the case of Hansen's skewed Student-t distribution, and Harvey and Siddique (1999) for the non-central Student-t. For all series, the parameters that capture skewness and kurtosis, φ_{01} and φ_{02}, are both significant at least at the one per cent level. The condition for the existence of the unconditional fourth moment of ε_t, given by ω_2 in (17), is met for all FX return series, whilst among the stock index series only Bovespa satisfies this condition. We also inspected the condition in (17) for the Normal-GJR and found it met for all series. These results show that when extending the Normal-GJR to the C-SNP-GJR in order to fit high-order moments, the condition for the existence of the unconditional fourth moment seems to become more demanding, in line with the results in Carnero, Peña and Ruiz (2004) for

[8] It is worth noting that we also considered v_{it} as function of past residuals, i.e. ε_t^+ and ε_t^-, and found it also worked well, and estimation results did not change significantly. Thus, we decided to work with the standardized residuals as they are more intuitively linked to skewness and kurtosis.

the case of the Student-t distribution. The last row of Table 2 presents the likelihood ratio (LR) test for the Normal-GJR and C-SNP-GJR. The LR test null is rejected for all series at any reasonable significance level, which shows that the SNP distribution significantly improves the Normal in fitting the skewness and leptokurtosis of the empirical return distributions.

Next, we analyze the goodness-of-fit of TV-SNP-GJR models with and without AR(1) component for $v_{i,t}$, equations (24) and (25) , respectively. Table 3 gives results for estimation of the TV1-SNP-GJR model. First, the parameter estimates of the conditional variance equation remain similar in magnitude to those of the C-SNP-GJR, and their statistical significance does not seem to be affected. Second, for all series, either φ_{2i}^{+} or φ_{2i}^{-} (or both) are significant. This implies all return series clearly present skewness and kurtosis clustering dynamics similar to that of volatility. Third, the asymmetric response of skewness and kurtosis to positive and negative shocks seems milder than that of volatility. Note that in $v_{1,t}$ equation, both φ_{21}^{+} and φ_{21}^{-} are statistically significant only for Nasdaq, UK-EU and JAP-US returns, whilst in $v_{2,t}$ equation, both φ_{22}^{+} and φ_{22}^{-} are significant for TAIEX, Bovespa, DAX and JAP-US returns.

Table 4 presents the results of estimating the TV2-SNP-GJR model. First, the conditional variance parameter estimate magnitudes are not affected by the addition of the AR(1) term to the $v_{i,t}$ equations. Second, for most series either φ_{11} or/and φ_{12} are significant indicating persistence in skewness and kurtosis. For UK-EU and JAP-US series, both φ_{11} and φ_{12} are not significant. Third, including φ_{1i} do not seem to alter much the estimates of φ_{2i}^{+} and φ_{2i}^{-}, whose magnitudes remain stable as can be clearly seen for TAIEX, CAC, EUROSTOXX and JAP-US returns. Fourth, for all series, either φ_{2i}^{+} or/and φ_{2i}^{-} remain statistically significant in relation to the results in Table 3. Fifth, note that φ_{21}^{+} and φ_{21}^{-} are both statistically significant only for CAC and JAP-US series, whilst φ_{22}^{+} and φ_{22}^{-} are both statistically significant only for Nasdaq, TAIEX and JAP-US returns. Summing up, for all series, the addition of the AR(1) term seems to provide a more parsimonious fit of the higher-order moment dynamics, as the TV2-SNP-GJR likelihood function value is larger than that of the TV1-SNP-GJR model.

Figure 1 presents plots and histograms of $skew_t$ and $Kurt_t$ series under the TV2-SNP-GJR specification for $v_{i,t}$, which are useful to corroborate that the model yields conditional skewness and kurtosis series within expected ranges. The average conditional skewness is -0.27 and -0.38 for CAC and JAP-US, respectively. These values should be compared with the sample skewness of the standardized residuals -0.19 and -0.23, respectively. Concerning the average conditional kurtosis, we find values of 3.81 for CAC and 4.39 for JAP-US, which compare to kurtosis of 4.40 and 5.77, respectively, from standardized residuals. In summary, our empirical statistics show predominant daily negative skewness and leptokurtosis typical of financial asset returns; see Albuquerque (2012) for a study on the negative skewness featured by index returns.

Finally, we implement both skewness and kurtosis NIC defined directly for $skew_t$ and $Kurt_t$, as well as for the parameters $v_{i,t}$, unlike the original NIC analysis of AP which is defined only for the parameters that determine the skewness. The NIC plots in Figure 2, specifically the linear asymmetric NIC in equation (22) for CAC and JAP-US returns (see rows 1 and 2 in Figure 2), show that both $skew_t$ and $Kurt_t$ (i) are highly non-linear functions of $v_{i,t}$, and (ii) respond all asymmetrically to positive and negative shocks. Besides, note that there is no sign asymmetry for skewness since it is negative whatever the sign of the shock. Regarding kurtosis NICs, we observe asymmetric response to either positive or negative shocks. For instance, the kurtosis NIC of CAC increases proportionally more after a positive rather than after a negative shock of the same magnitude. We implement transition NIC in equation (23), as well as asymmetric NIC, for the case of TAIEX (see rows 3 and 4) in order to study the performance under an alternative NIC specification. The transition NICs display similar but smoother patterns as the asymmetric NICs for both skewness and kurtosis.

Table 2: C-SNP-GJR model estimation results

	Nasdaq	TAIEX	Bovespa	CAC	DAX	EUROSTOXX	UK-EU	JAP-US	CAN-US	UK-US
μ	0.0247	0.0242	0.0312	0.0192	0.0338**	0.0160	0.0009	0.0039	0.0039	-0.0027
	(0.0151)	(0.0155)	(0.0273)	(0.0162)	(0.0167)	(0.0163)	(0.0060)	(0.0085)	(0.0062)	(0.0069)
α_0	0.0168*	0.0072**	0.1262*	0.0304*	0.0294*	0.0307*	0.0011**	0.0051*	0.0010*	0.0026**
	(0.0040)	(0.0034)	(0.0348)	(0.0075)	(0.0073)	(0.0078)	(0.0005)	(0.0018)	(0.0004)	(0.0010)
β	0.9201*	0.9534*	0.8992*	0.9083*	0.9193*	0.9122*	0.9571*	0.9510*	0.9527*	0.9454*
	(0.0088)	(0.0091)	(0.0153)	(0.0126)	(0.0087)	(0.0119)	(0.0083)	(0.0091)	(0.0047)	(0.0142)
α_1^+	0.0169*	0.0208*	0.0287*	0.0165**	0.0118	0.0125***	0.0366*	0.0319*	0.0520*	0.0532*
	(0.0065)	(0.0061)	(0.0079)	(0.0076)	(0.0081)	(0.0067)	(0.0064)	(0.0068)	(0.0074)	(0.0123)
α_1^-	0.1255*	0.0649*	0.1248*	0.1398*	0.1224*	0.1359*	0.0430*	0.0433*	0.0373*	0.0414**
	(0.0137)	(0.0122)	(0.0195)	(0.0195)	(0.0142)	(0.0190)	(0.0132)	(0.0097)	(0.0050)	(0.0182)
φ_{01}	-0.9014*	0.5788*	0.5981*	0.5526*	0.5466*	0.5849*	0.6468*	0.6716*	0.5964*	0.6486*
	(0.0341)	(0.0266)	(0.0434)	(0.0565)	(0.0537)	(0.0590)	(0.0414)	(0.0295)	(0.0450)	(0.0380)
φ_{02}	0.3249*	0.2870*	0.2804*	0.2231*	0.2223*	0.2342*	0.2348*	0.3349*	0.2172*	0.2649*
	(0.0295)	(0.0195)	(0.0282)	(0.0315)	(0.0295)	(0.0349)	(0.0263)	(0.0242)	(0.0294)	(0.0279)
$\sigma_{SNP-GJR}$	1.6860	1.9791	2.4560	1.6735	1.6262	1.6361	0.5827	0.6828	0.6135	0.5860
ω_2	1.0112	1.0070	0.9954	1.0154	1.0066	1.0117	0.9975	0.9838	0.9992	0.9908
Λ	52.36	161.27	156.68	77.74	78.85	77.74	25.92	220.94	30.71	55.83

Model: $r_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t(\theta)\, z_t, \quad \sigma_t^2 = \alpha_0 + \beta\sigma_{t-1}^2 + \alpha_1^+(\varepsilon_{t-1}^+)^2 + \alpha_1^-(\varepsilon_{t-1}^-)^2, \quad v_i = \varphi_{0i}, \quad i = 1,2, \quad z_t \sim iid\ g(z_t; v)$.

This table presents ML estimates of the parameters of the C-SNP-GJR model (skewness and kurtosis are specified as constants in this model) for stock index and FX rate percent log returns (sample: 5,218 obs.). Heteroscedasticity-consistent standard errors are provided in parentheses below the parameter estimates. (*) indicates significance at 1% level; (**) indicates significance at 5% level and (***) indicates significance at 10% level. $\sigma_{SNP-GJR}$ denotes the implied unconditional standard deviation obtained from the model estimates. $\Lambda = 2(LL_1 - LL_0)$ denotes likelihood ratio statistic where LL_0 and LL_1 give log likelihood values (constant terms included) of models Normal-GJR and C-SNP-GJR, respectively. ω_2 denotes the condition for the existence of the unconditional fourth moment in (17). In this case, Λ is χ^2 asymptotically distributed with two degrees of freedom, the critical value of the χ_2^2 for significance level 5% is 5.99.

Table 3: TV1-SNP-GJR model estimation results

	Nasdaq	TAIEX	Bovespa	CAC	DAX	EUROSTOXX	UK-EU	JAP-US	CAN-US	UK-US
μ	0.0384**	0.0203	0.0204	0.0160	0.0034	0.0102	-0.0075	0.0041	0.0026	-0.0064
	(0.0154)	(0.0158)	(0.0274)	(0.0163)	(0.0152)	(0.0163)	(0.0056)	(0.0085)	(0.0061)	(0.0069)
α_0	0.0157*	0.0080**	0.1279*	0.0313*	0.0210*	0.0327*	0.0013*	0.0054*	0.0008*	0.0021*
	(0.0046)	(0.0039)	(0.0338)	(0.0077)	(0.0049)	(0.0082)	(0.0001)	(0.0019)	(0.0003)	(0.0003)
β	0.9250*	0.9509*	0.8938*	0.9075*	0.9115*	0.9077*	0.9574*	0.9473*	0.9565*	0.9573*
	(0.0125)	(0.0107)	(0.0158)	(0.0128)	(0.0086)	(0.0125)	(0.0016)	(0.0099)	(0.0036)	(0.0032)
α_1^+	0.0137***	0.0249*	0.0415*	0.0187**	0.0105	0.0189*	0.0327*	0.0341*	0.0497*	0.0361*
	(0.0073)	(0.0074)	(0.0099)	(0.0076)	(0.0069)	(0.0071)	(0.0018)	(0.0072)	(0.0052)	(0.0027)
α_1^-	0.1199*	0.0677*	0.1337*	0.1416*	0.1611*	0.1415*	0.0427*	0.0475*	0.0335*	0.0365*
	(0.0178)	(0.0142)	(0.0204)	(0.0196)	(0.0153)	(0.0196)	(0.0023)	(0.0106)	(0.0033)	(0.0058)
φ_{01}	-0.5504*	0.5487*	0.5217*	0.4934*	0.6863*	0.4623*	0.6141*	0.5390*	0.6364*	0.6845*
	(0.1639)	(0.0427)	(0.0576)	(0.0668)	(0.0591)	(0.0805)	(0.0374)	(0.0677)	(0.0617)	(0.0663)
φ_{21}^+	0.1760*	0.1130**	0.1947*	0.1801*	0.0287	0.2165*	-0.6849*	0.1816**	-0.4863*	-0.0484
	(0.0574)	(0.0542)	(0.0526)	(0.0560)	(0.0400)	(0.0640)	(0.0407)	(0.0770)	(0.0316)	(0.0449)
φ_{21}^-	0.2227*	-0.0102	-0.0863	0.0568	0.1891**	-0.0613	0.1651*	-0.1113**	0.0458	-0.1613**
	(0.0749)	(0.0468)	(0.0823)	(0.0448)	(0.0763)	(0.0584)	(0.0398)	(0.0436)	(0.0300)	(0.0770)
φ_{02}	0.0948	0.2331*	0.1960*	0.1583*	0.1362*	0.1293*	0.1654*	0.2335*	0.2260*	0.2911*
	(0.0922)	(0.0266)	(0.0386)	(0.0392)	(0.0394)	(0.0461)	(0.0189)	(0.0449)	(0.0398)	(0.0522)
φ_{22}^+	-0.2045*	0.1615*	0.2631*	0.1968*	0.2237*	0.2412*	-0.2521*	0.1341*	-0.1950*	-0.0295
	(0.0346)	(0.0421)	(0.0441)	(0.0446)	(0.0344)	(0.0484)	(0.0158)	(0.0516)	(0.0277)	(0.0215)
φ_{22}^-	-0.1105	-0.0554**	-0.0859***	-0.0154	-0.0618***	-0.0633	0.0051	-0.0996*	-0.0009	0.1688*
	(0.0861)	(0.0254)	(0.0520)	(0.0338)	(0.0363)	(0.0458)	(0.0260)	(0.0373)	(0.0330)	(0.0509)
LL	-8522.92	-8678.41	-11291	-8812.36	-9038.31	-8867.20	-3582.56	-4986	-3684.79	-4078.31

Model: $r_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t(\theta)\, z_t, \quad \sigma_t^2 = \alpha_0 + \beta\sigma_{t-1}^2 + \alpha_1^+(\varepsilon_{t-1}^+)^2 + \alpha_1^-(\varepsilon_{t-1}^-)^2, \quad v_{i,t} = \varphi_{0i} + \varphi_{2i}^+(z_{t-1}^+)^2 + \varphi_{2i}^-(z_{t-1}^-)^2, \quad i = 1,2, \quad z_t \sim iid\; g(z_t; v_t)\,.$

This table presents ML estimates of the parameters of the TV1-SNP-GJR model for stock index and FX rate percent log returns (sample: 5,218 obs.). Heteroscedasticity-consistent standard errors are provided in parentheses below the parameter estimates. (*) indicates significance at 1% level; (**) indicates significance at 5% level and (***) indicates significance at 10% level. LL gives log likelihood values (constant terms included) of the model.

Table 4: TV2-SNP-GJR model estimation results

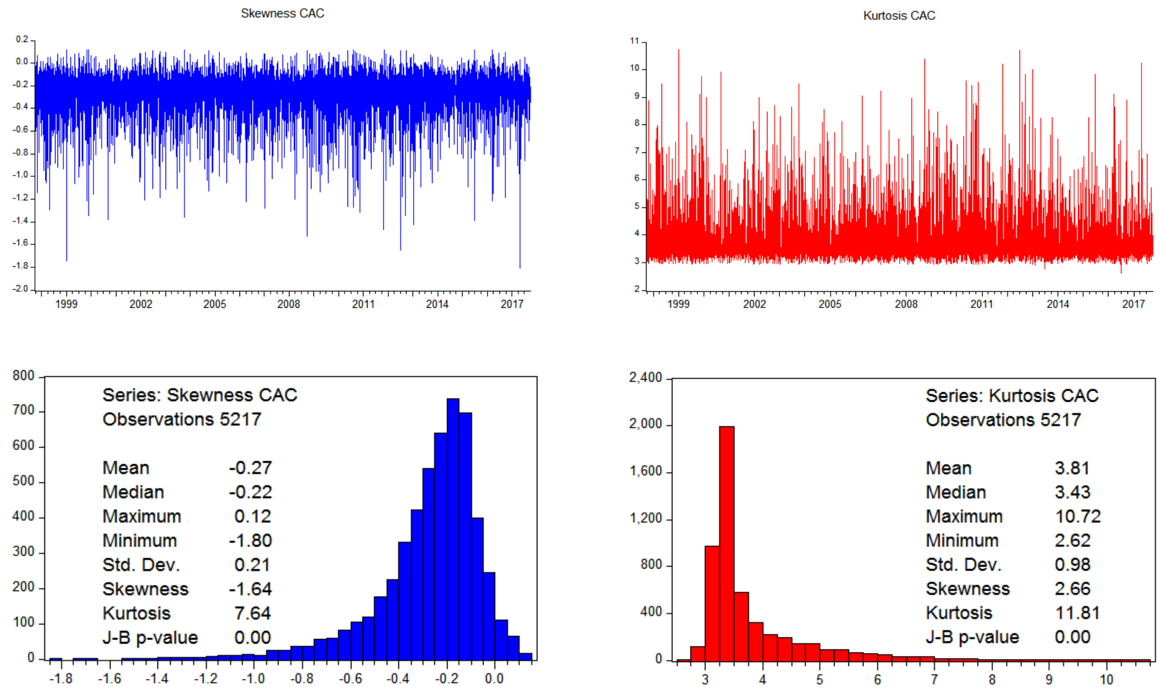
	Nasdaq	TAIEX	Bovespa	CAC	DAX	EUROSTOXX	UK-EU	JAP-US	CAN-US	UK-US
μ	0.0075	0.0199	0.0096	0.0163	0.0476*	0.0109	-0.0006	0.0036	0.0047	-0.0048
	(0.0172)	(0.0164)	(0.0281)	(0.0162)	(0.0153)	(0.0167)	(0.0062)	(0.0086)	(0.0062)	(0.0072)
α_0	0.0093*	0.0087**	0.1107*	0.0299*	0.0242*	0.0327*	0.0010**	0.0053*	0.0009*	0.0034**
	(0.0035)	(0.0040)	(0.0288)	(0.0071)	(0.0058)	(0.0081)	(0.0005)	(0.0018)	(0.0003)	(0.0015)
β	0.9364*	0.9503*	0.9017*	0.9092*	0.9140*	0.9076*	0.9577*	0.9485*	0.9544*	0.9365*
	(0.0099)	(0.0109)	(0.0141)	(0.0121)	(0.0087)	(0.0126)	(0.0079)	(0.0097)	(0.0039)	(0.0193)
α_1^+	0.0160***	0.0252*	0.0430*	0.0181**	0.0220*	0.0186*	0.0355*	0.0339*	0.0509*	0.0596*
	(0.0086)	(0.0081)	(0.0100)	(0.0074)	(0.0071)	(0.0070)	(0.0062)	(0.0074)	(0.0059)	(0.0162)
α_1^-	0.1087*	0.0687*	0.1245*	0.1396*	0.1307*	0.1421*	0.0431*	0.0456*	0.0357*	0.0491**
	(0.0154)	(0.0142)	(0.0176)	(0.0188)	(0.0129)	(0.0198)	(0.0128)	(0.0102)	(0.0040)	(0.0223)
φ_{01}	0.1902	0.1614**	0.0807**	0.6151*	0.4052*	0.5724*	0.5557**	0.7811*	0.4182*	0.5258*
	(0.1776)	(0.0629)	(0.0397)	(0.1135)	(0.0765)	(0.1254)	(0.2621)	(0.1986)	(0.0688)	(0.0986)
φ_{11}	0.6400**	0.6181*	0.7949*	-0.2611***	0.1118	-0.2802**	0.3046	-0.3650	0.3813*	0.0413
	(0.3178)	(0.1143)	(0.0778)	(0.1546)	(0.1036)	(0.1138)	(0.2995)	(0.2421)	(0.0883)	(0.1113)
φ_{21}^+	0.0343	0.1523*	0.1085*	0.1909*	0.2462*	0.2485*	-0.0811	0.1615***	-0.4306*	0.1771*
	(0.0571)	(0.0459)	(0.0346)	(0.0626)	(0.0547)	(0.0830)	(0.0898)	(0.0865)	(0.0346)	(0.0384)
φ_{21}^-	0.1127*	-0.0239	-0.0233	0.0581***	0.0573	-0.0902	0.2260*	-0.1212*	0.0031	-0.0623
	(0.0409)	(0.0324)	(0.0266)	(0.0339)	(0.0409)	(0.0819)	(0.0419)	(0.0398)	(0.0352)	(0.0440)
φ_{02}	0.0346***	0.0713**	0.0525**	0.2227*	-0.0905**	0.1444***	0.1252	0.3241*	0.1316*	0.0816**
	(0.0197)	(0.0332)	(0.0228)	(0.0754)	(0.0372)	(0.0810)	(0.3263)	(0.1017)	(0.0259)	(0.0319)
φ_{12}	0.7976*	0.5041*	0.5924*	-0.3403	0.2253*	-0.1706	0.6648	-0.2594	0.4629*	0.2610*
	(0.0751)	(0.1278)	(0.0621)	(0.2626)	(0.0568)	(0.2237)	(1.1119)	(0.1980)	(0.0750)	(0.0907)
φ_{22}^+	0.0983*	0.1866*	0.1962*	0.2011*	0.4237*	0.2622*	-0.0553	0.1256**	-0.1636*	0.1847*
	(0.0324)	(0.0379)	(0.0324)	(0.0497)	(0.0413)	(0.0595)	(0.1065)	(0.0553)	(0.0252)	(0.0323)
φ_{22}^-	0.0512**	-0.0506***	-0.0305	-0.0215	-0.1799*	-0.0791	0.0672*	-0.1030*	-0.0176	-0.1256*
	(0.0235)	(0.0295)	(0.0269)	(0.0323)	(0.0296)	(0.0562)	(0.0237)	(0.0351)	(0.0335)	(0.0373)
LL	-8445.15	-8672.86	-11283.18	-8810.87	-9014.14	-8866.2	-3473.61	-4984.87	-3675.17	-4023.93

Model: $r_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t(\theta) z_t, \quad \sigma_t^2 = \alpha_0 + \beta\sigma_{t-1}^2 + \alpha_1^+(\varepsilon_{t-1}^+)^2 + \alpha_1^-(\varepsilon_{t-1}^-)^2, \quad v_{i,t} = \varphi_{0i} + \varphi_{1i} v_{i,t-1} + \varphi_{2i}^+(z_{t-1}^+)^2 + \varphi_{2i}^-(z_{t-1}^-)^2, \quad i = 1, 2, \quad z_t \sim iid\ g(z_t; v_t)$.

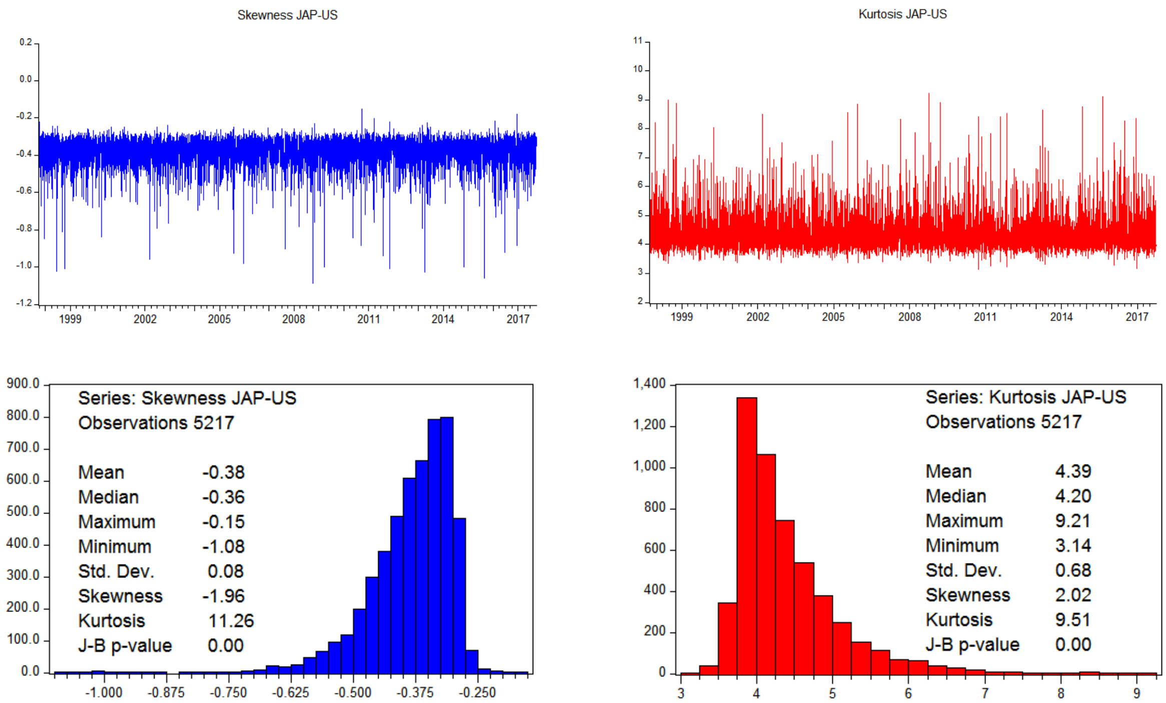
This table presents ML estimates of the parameters of the TV2-SNP-GJR model for stock index and FX rate percent log returns (sample: 5,218 obs.). Heteroscedasticity-consistent standard errors are provided in parentheses below the parameter estimates. (*) indicates significance at 1% level; (**) indicates significance at 5% level and (***) indicates significance at 10% level. LL gives log likelihood values (constant terms included) of the model.

Figure 1: Conditional skewness and kurtosis

CAC

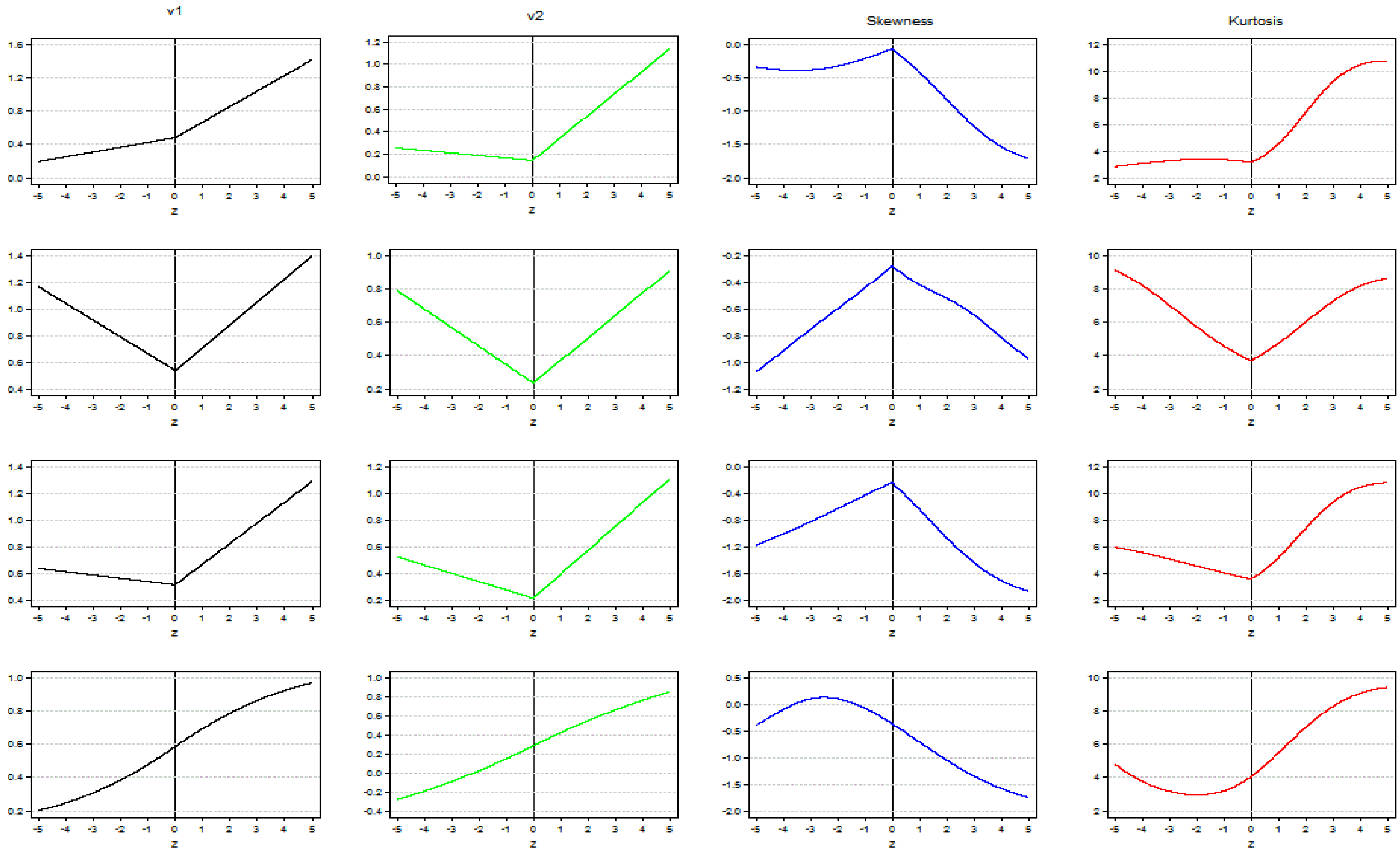


JAP-US



Plots and histograms of conditional skewness (blue) and kurtosis (red) from model TV2-SNP-GJR. Return series: CAC (upper panel), JAP-US (lower panel).

Figure 2: Skewness and kurtosis NIC



Plots of v_{1t} (black), v_{2t} (green), skewness (blue) and kurtosis (red) NIC from asymmetric (rows 1 to 3) and transition (row 4) specifications. Return series: CAC (row 1), JAP-US (row 2), TAIEX (rows 3 and 4). Model: TV2-SNP-GJR.

4 Conditional performance measures

The PMs we use in this section have been selected in order to include the well-known Sharpe ratio (Sharpe, 1966, 1994) as well as measures based on higher moments, partial moments and quantiles. Some references, among others, Caporin, Jannin, Lisi and Maillet (2014), Bacon (2008) and Eling and Schuhmacher (2007), might be a good starting guide to the large set of PMs proposed in the financial economics literature. Next, we consider PMs with closed-form expressions under the conditional TV-SNP-GJR distribution. In short, we implement conditional parametric PMs.

4.1 Sharpe ratio

We start with the SR as our benchmark PM. A slightly different version of the SR is defined as $(\mu_t - \theta)/\sigma_t$, where θ is the return threshold (e.g., risk-free rate, zero return,...), $\mu_t \equiv E[r_t | I_{t-1}]$ and $\sigma_t \equiv \sqrt{V[r_t | I_{t-1}]}$ denote the conditional mean and volatility of the stock return. A drawback of using the previous ratio for ranking stocks occurs when the numerator, or excess return $\mu_t - \theta$, is negative. This pattern is very common in bear markets (high level of recession and unemployment) like the most recent U.S. bear market occurred in 2007-2009. Israelsen (2005) suggests a modified version to overcome that problem. It consists of adding an exponent to the denominator of SR, i.e. the exponent is the *sign* function of the numerator. In short, our SR onwards is as follows:

$$SR_t(\theta) = \frac{\mu_t - \theta}{\sigma_t^{sgn(\mu_t - \theta)}}, \tag{36}$$

where $sgn(z) = z/|z|$ if $z \neq 0$ and $sgn(z) = 0$ if $z = 0$.

4.2 Skewness-kurtosis ratio

Pézier and White (2006) suggests using an adjusted SR which explicitly adjusts for skewness and kurtosis by incorporating a penalty factor for both negative skewness and excess kurtosis. This measure has been welcomed by practitioners since it does take into account these higher moments. Nevertheless, a recent study by León, Navarro and Nieto (2018) shows that when ranking stocks from the S&P 500 index there is no barely difference between this PM and the SR. An alternative PM, suggested by Watanabe (2006), aims to explicitly adjust for skewness and kurtosis by using the simple skewness-kurtosis ratio, $s_{r,t}/k_{r,t}$, where $s_{r,t} \equiv skew(r_t | I_{t-1})$ and $k_{r,t} \equiv kur(r_t | I_{t-1})$ denote, respectively, the conditional skewness and kurtosis of the stock return in (20). Again, higher rather than lower ratios are preferred. Since this PM may lead to ranking problems when the numerator becomes negative, we propose –as a possible solution– a modified version based on Israelsen's aforementioned idea. Hereafter, our skewness-kurtosis ratio is defined as

$$SKR_t = \frac{s_{r,t}}{k_{r,t}^{sgn(s_{r,t})}}. \tag{37}$$

4.3 PMs based on partial moments

In this section we obtain conditional parametric PMs based on the SNP distribution. Unconditional parametric PMs based on partial moments under the Gram-Charlier and the SNP distributions can be found in León and Moreno (2017). The Sortino ratio (Sortino and van der Meer, 1991) is the mean excess return, $\mu_t - \theta$, per unit of risk measured by the square root of LPM of order 2 of the stock return defined in (34). Note that this PM presents the same problem as the previous measures since the numerator may be negative. As a solution, we propose the following modified Sortino ratio:

$$Sortino_t(\theta) = \frac{\mu_t - \theta}{\left(\sqrt{LPM_t(\theta, 2)}\right)^{sgn(\mu_t - \theta)}}. \tag{38}$$

Next, we use two PMs which are special cases of the Farinelli and Tibiletti (2008) ratio:

$$FT_t(\theta, q, m) = \frac{\sqrt[q]{UPM_t(\theta, q)}}{\sqrt[m]{LPM_t(\theta, m)}}, \tag{39}$$

with $q > 0$ and $m > 0$. The higher the value for q, the greater the investor's preference for expected gain, and the higher the value for m the greater the investor's dislike of expected losses. If $q = m = 1$, we have the Omega ratio (Keating and Shadwick, 2002) and for $q = 1$ and $m = 2$, we have the Upside potential ratio (Sortino, van der Meer and Platinga, 1999). These PMs will be represented as $FT_t(\theta, 1, 1)$ and $FT_t(\theta, 1, 2)$, respectively.

4.4 PMs based on quantiles

A class of PMs similar to (39) replaces UPM and LPM, respectively, with reward and risk measures based on quantiles. First, the VaRR (Caporin and Lisi, 2011), defined as the ratio of the upper and lower quantiles of the stock return distribution, is given by

$$VaRR_t(\alpha) = \left| \frac{VaR_t(1-\alpha)}{VaR_t(\alpha)} \right|, \tag{40}$$

where $VaR_t(\alpha) \equiv Q_t^{-1}(\alpha)$ and $VaR_t(1-\alpha) \equiv Q_t^{-1}(1-\alpha)$ are, respectively, the conditional lower and upper quantiles of r_t given the information set I_{t-1} with α set equal to 1%, 5%, 10% and 20%. See equation (29).

Second, the ETR or Rachev ratio (Biglova et al., 2004) is defined as

$$ETR_t(\alpha) = \left| \frac{ES_t(-r_t, \alpha)}{ES_t(r_t, \alpha)} \right|, \tag{41}$$

where $ES_t(r_t, \alpha)$ is just the conditional ES measure in (30) with r_t as the random variable, while $ES_t(-r_t, \alpha)$ is the same definition but replacing r_t with $-r_t$. Thus, the numerator is the reward measure corresponding to the right-hand side (gains) of the return distribution, $E_{t-1}(r_t \,|r_t \geq VaR_t(1-\alpha))$, while the denominator is the risk measure defined as $E_{t-1}(r_t \,|r_t \leq VaR_t(\alpha))$. We can rewrite (40), similar to (41), as a quotient of conditional lower quantiles, i.e. $VaR_t(\alpha) \equiv VaR_t(r_t, \alpha)$ and $VaR_t(1-\alpha) \equiv VaR_t(-r_t, \alpha)$. An application of (41) for portfolio selection is provided by Bruni, Cesarone, Scozzari and Tardella (2017)

5 Equity screening and portfolio selection

Once we have proposed the SNP distribution for stock returns and discussed its properties and estimation, we now apply the TV-SNP-GJR pdf for equity screening under alternative PMs to create portfolios.

5.1 Dataset description

We study the performance of portfolios formed from choosing stocks that were constituents of the S&P 100 index in October 2017. The data series used are sampled over the period November 4, 2004 to October 18, 2017, a total of $T = 3,262$ daily log-return observations. After filtering, we restrict to 90 stocks that continuously belonged to S&P 100 during our sample period. We split the series into two subsamples, one for in-sample and another for the out-of-sample (OOS) period. The in-sample period goes from November 4, 2004 to December 7, 2009. We always use a constant-size rolling window of $N_w = 1,282$ for the in-sample and, also, when estimating across the OOS period. The stocks in our study are listed in Table 5, along with their ticker symbol and three-digit Standard Industrial Classification (SIC) codes.

Table 6 presents some summary statistics of data analyzed in this section. The top panel presents sample moments only for the in-sample daily returns. The means and standard deviations are typical of asset returns. The kurtosis coefficients reveal the stock return distributions are highly leptokurtic (kurtosis median is 10.883). In contrast with the stock indexes in Table 1, the skewness of the single stocks is predominantly positive (skewness median is 0.0687).

Table 5: S&P 100 stocks used in the empirical analysis

Ticker	Name	SIC	Ticker	Name	SIC	Ticker	Name	SIC
MMM	3M	384	COST	Costco Wholesale	533	MS	Morgan Stanley	621
ABT	Abbott Laboratories	283	CVS	CVS Health	591	NEE	Nextera Energy	491
CAN	Accenture Class A	738	DHR	Danaher	382	NKE	Nike 'B'	302
AGN	Allegan	283	DUK	Duke Energy	493	OXY	Occidental Ptl.	131
ALL	Allstate	633	LLY	Eli Lilly	283	ORCL	Oracle	737
GOOGL	Alphabet 'A'	737	EMR	Emerson Ellectric	360	PEP	Pepsico	208
MO	Altria Group	211	EXC	Exelon	493	PFE	Pfizer	283
AMZN	Amazon.com	737	XOM	Exxon Mobil	291	PCLN	Priceline Group	738
AXP	American Express	671	FDX	Fedex	451	PG	Procter & Gamble	284
AIG	American Intl. GP.	633	F	Ford Motor	371	QCOM	Qualcomm	366
AMGN	Amgen	283	GD	General Dynamics	373	RTN	Raytheon 'B'	381
AAPL	Apple	357	GE	General Electric	360	SLB	Schlumberger	138
T	AT&T	481	GILD	Gilead Sciences	283	SPG	Simon Property Group	679
BAC	Bank of America	602	GS	Goldman Sachs GP.	621	SO	Southern	491
BK	Bank of New York Mellon	602	HAL	Halliburton	138	SBUX	Starbucks	581
BRK	Berskshire Hathaway 'B'	633	HD	Home Depot	521	TGT	Target	533
BIIB	Biogen	283	HON	Honeywell Intl.	371	TXN	Texas Instruments	367
BLK	Blackrock	621	INTC	Intel	367	TWX	Time Warner	484
BAC	Boeing	372	IBM	International Bus. Mchs.	357	FOXA	Twenty-First Century Fox Cl. B	484
BMY	Bristol Myers Squibb	283	JNJ	Johnson & Johnson	283	FOX	Twenty-First Century Fox Cl. A	271
COF	Capital One Finl.	602	JPM	JP Morgan Chase & Co.	602	UNP	Union Pacific	401
CAT	Caterpillar	353	LMT	Lokheed Martin	376	UPS	United Parcel Ser 'B'	421
CELG	Celgene	283	LOW	Lowe's Companies	521	UTX	United Technologies	372
CVX	Chevron	291	MCD	McDonalds	581	UNH	Unitedhealth Group	632
CSCO	Cisco Systems	357	MDT	Medtronic	384	UNB	US Bankcorp	602
C	Citigroup	602	MRK	Merck & Company	283	VZ	Verizon Communications	481
KO	Coca Cola	208	MET	Metlife	631	WMT	Wal Mart Stores	533
CL	Colgate-Palm.	284	MSFT	Microsoft	737	WBA	Walgreens Boots Alliance	591
CMCSA	Comcast 'A'	484	MDLZ	Mondelez International Cl. A	200	DIS	Walt Disney	484
COP	Conocophillips	291	MON	Monsanto	287	WFC	Wells Fargo & Co	602
	Description	Num		Description	Num		Description	Num
SIC 1	Mining, constuct.	3	SIC 4	Transprt, comm's	13	SIC 7	Services	6
SIC 2	Manuf: food, furn.	22	SIC 5	Trade	9			
SIC 3	Manuf: elec, mach.	20	SIC 6	Finance, Ins.	17			

This table presents the ticker, names, and three-digit Standard Industrial Classification (SIC) of 90 stocks that constitute the S&P 100 index used in the empirical analysis of this article. The lower panel reports the number of stocks in each one-digit SIC group.

Table 6: Summary statistics of S&P 100 stocks and estimation results

	Mean	5%	25%	Median	75%	95%	M
			Cross-sectional distribution				
			In-sample period: 11/4/2004-12/7/2009				
Daily obs. 1,282							
Mean	0.0096	-0.0539	-0.0142	0.0129	0.0290	0.0983	
Std. dev.	2.3180	1.3097	1.6967	2.0569	2.6879	4.1054	
Skewness	-0.0185	-0.7673	-0.2129	0.0687	0.2832	0.8368	
Kurtosis	13.936	7.1903	8.6502	10.883	14.893	24.162	
Conditional mean							
γ_0	0.0316	-0.0264	0.0011	0.0242	0.0511	0.1258	0.03
γ_1	0.0104	-0.0938	-0.0240	0.0125	0.0519	0.0963	0.07
γ_m	-0.1138	-0.2404	-0.1521	-0.1256	-0.0667	0.0180	0.31
Conditional variance							
α_0	0.1580	0.0162	0.0439	0.0739	0.1259	0.5767	0.42
β	0.8750	0.7238	0.8556	0.8884	0.9156	0.9529	0.76
α_1^+	0.0232	0.0000	0.0000	0.0131	0.0374	0.0713	0.11
α_1^-	0.0683	0.0000	0.0256	0.0622	0.0942	0.1816	0.37
δ_m^+	0.0354	0.0000	0.0000	0.0000	0.0240	0.1763	0.09
δ_m^-	0.1765	0.0202	0.0697	0.1277	0.2027	0.5329	0.42
C-SNP							
φ_{01}	0.7477	0.4056	0.6475	0.7242	0.8068	1.2177	1
φ_{02}	0.3468	0.1704	0.2705	0.3345	0.3982	0.6182	0.98
TV1-SNP							
φ_{01}	0.7325	0.2018	0.6139	0.7108	0.8404	1.2597	0.96
φ_{21}^+	0.0034	-0.5236	-0.1848	0.0291	0.1897	0.4588	0.66
φ_{21}^-	-0.0194	-0.6294	-0.1953	0.0258	0.1822	0.4608	0.57
φ_{02}	0.3549	0.0124	0.2460	0.3446	0.4812	0.6285	0.88
φ_{22}^+	-0.0540	-0.4200	-0.1731	-0.0522	0.0428	0.3021	0.53
φ_{22}^-	-0.0379	-0.4220	-0.1458	0.0062	0.0826	0.2001	0.37

Model (TV1-SNP-GJRA):

$$r_{j,t} = \gamma_{0j} + \gamma_{1j} r_{j,t-1} + \gamma_{mj} r_{m,t-1} + \varepsilon_{j,t}, \quad \varepsilon_{j,t} = \sigma_{j,t}(\theta) z_{j,t}, \quad j = 1, \ldots, 90, \quad z_{j,t} \sim iid\; g(z_{j,t}; v_{j,it}),$$

$$\sigma^2_{j,t} = \alpha_{0j} + \beta_j \sigma^2_{j,t-1} + \alpha^+_{1j}\left(\varepsilon^+_{j,t-1}\right)^2 + \alpha^-_{1j}\left(\varepsilon^-_{j,t-1}\right)^2 + \delta^+_{mj}\left(\varepsilon^+_{m,t-1}\right)^2 + \delta^-_{mj}\left(\varepsilon^-_{m,t-1}\right)^2,$$

$$v_{j,it} = \varphi_{0ij} + \varphi^+_{2ij}\left(z^+_{j,t-1}\right)^2 + \varphi^-_{2ij}\left(z^-_{j,t-1}\right)^2, \quad i = 1, 2.$$

This table presents some summary statistics of the in-sample daily log returns of stocks that constitute the S&P 100 index data used in this analysis. The columns present the mean, median and percentiles from the cross-sectional distribution of the measures listed in the rows. The second panel presents summaries of the estimated C-SNP-GJRA ($\varphi^+_{2ij} = \varphi^-_{2ij} = 0$) and TV1-SNP-GJRA models for the first in-sample window returns, j denotes an individual stock from S&P 100, and M denotes the number of stocks out of 90 (in %) with significant parameter estimates at 5% level.

5.2 Model and estimation of individual stock returns

The procedure to estimate all the parameters for each stock return series, $r_{j,t}$, is based now on the Gaussian quasi-ML (QML) method. Thus, at a first stage, we estimate the conditional mean and variance by using the specification from Oh and Patton (2017). Specifically, the AR(1)-GJR(1,1) model augmented with lagged market (S&P 100) return information for the stock return series is given by:

$$r_{j,t} = \gamma_{0j} + \gamma_{1j} r_{j,t-1} + \gamma_{mj} r_{m,t-1} + \varepsilon_{j,t}, \qquad \varepsilon_{j,t} = \sigma_{j,t} z_{j,t}, \qquad j = 1, ..., 90 \tag{42}$$

$$\sigma^2_{j,t} = \alpha_{0j} + \beta_j \sigma^2_{j,t-1} + \alpha^+_{1j} \left(\varepsilon^+_{j,t-1}\right)^2 + \alpha^-_{1j} \left(\varepsilon^-_{j,t-1}\right)^2 + \delta^+_{mj} \left(\varepsilon^+_{m,t-1}\right)^2 + \delta^-_{mj} \left(\varepsilon^-_{m,t-1}\right)^2, \tag{43}$$

where $\varepsilon_{m,t}$ is the demeaned market return. Onwards, we refer to model in (42)-(43) as GJRA. In the lower panel of Table 6 we present information on the parameter estimates of the model above for the in-sample period. The estimation is carried out in two stages. Thus, given the first-stage (Gaussian) QML estimation in which we obtain the standardized returns, i.e. $z_{j,t} = \varepsilon_{j,t}/\sigma_{j,t}$, the second stage consists of estimating the parameters of alternative specifications of the SNP distribution by ML. Our estimates of the mean equation show a small positive AR(1) coefficient, γ_{1j}, that is significant only for 7% of the stocks, and an estimate for the lagged market return parameter, γ_{mj}, that is predominantly negative, larger in magnitude than γ_{1j} and significant for 31% of the stocks. The conditional variance parameter estimates in (43) show most stock returns exhibit typical volatility clustering and high persistence in volatility, as well as asymmetric response of volatility to positive and negative news. Furthermore, stock volatility asymmetric response is in average greater to market shocks than to individual ones. We find evidence of leverage effect since the average estimate of α^-_{1j} is higher than that of α^+_{1j}, namely $0.06873 > 0.0232$. The second-stage estimates for the C-SNP model show that virtually all stock returns present asymmetric and leptokurtic distributions. Besides, the estimated TV1-SNP parameters show evidence of larger response of v_{2t} to positive rather than to negative shocks, whilst the response of v_{1t} to shocks is more symmetric (see median values).

5.3 Time-varying portfolio selection

Through our constant-size rolling window, we obtain the estimations of a battery of PMs across the OOS period for each individual stock and setting a zero mean return as the threshold, $\theta = 0$. We compute a total of thirteen PMs, namely: SR in (36), SKR in (37), Sortino (38), Omega and Upside potential ratios nested within the FT family in (39), as well as VaRR in (40) and ETR in (41) for the levels of α: 1%, 5%, 10% and 20%.

Next, we explain the steps to construct the different portfolios. First, the last day of each window, we compute all PMs based on the one-day-ahead forecast of the conditional mean, variance and $v_{i,t}$ for each stock assuming the TV1-SNP specification for $z_{j,t}$ in (42). Second, the stocks are ranked on the basis of each PM and then, we select the ten best-ranked stocks to build initially an equally-weighted (EW) portfolio, i.e. $w_{j,t} = 1/10$ where $j = 1, ..., 10$. We keep this portfolio for the next 5 days to then, compute the daily portfolio returns for these five days. Third, by rolling the window each five days, we repeat the previous two steps a total of 396 times and change each time the portfolio composition according to the equity screening from the different PMs. Fourth, we obtain thirteen OOS portfolio return series of $1,980$ daily observations. We denote each of these return series according to the selected PM.

We also repeat the above procedure but changing now the rebalancing frequency. So, we estimate each stock return model under the OOS period every 22 days (monthly frequency) and 10 days (biweekly frequency). Thus, these two rebalancing horizons account for 90 and 198 estimations, respectively. We estimate both the C-SNP-GJRA and TV1-SNP-GJRA models for all S&P 100 stocks.[9] Hereafter, we only use the latter model to build the alternative PMs.

[9] For the second estimation stage, if we estimate the TV-SNP with the AR component, we can find identification problems when $\varphi^+_{2i} = \varphi^-_{2i} = 0$ since there exists a parameter subset $(\varphi_{0i}, \varphi_{1i})$ verifying $(1 - \varphi_{1i})\, v^*_i = \varphi_{0i}$, where v^*_i denotes the stationary level. More details can be seen in the Monte-Carlo simulation in Appendix C in JR. Note the high number of estimations involved in our application, e.g. $35,640$ for weekly rebalancing, makes rather impracticable to check for identification problems. This is

Figure 3 represents the spreads between the cumulative returns on each portfolio and the SR under the TV1-SNP-GJRA during the OOS period for the three different rebalancing periods. It is exhibited that the size of spreads –notice the scale in the vertical axis– become much higher under both SKR and ETRs, except for the ETR (95,5). Negative spreads, displayed the majority of days, are obtained under monthly frequencies in many portfolios. We also find that VaRR portfolios show positive spreads in most cases under biweekly frequency except for the VaRR (80,20) where, surprisingly, the monthly frequency cumulative returns are consistently higher. Finally, unlike the Omega portfolio, we obtain positive spreads under both Sortino and Upside potential portfolios for weekly frequency.

A similar analysis is carried out in Figure 4 but the spread is now respecting the S&P 100 index returns. Again, the portfolios with the best performance correspond to both ETR and SKR strategies. The monthly rebalancing yields lower performance although rather better than in the case displayed in Figure 3.[10]

We also compute the portfolio turnover, not presented here to save space, as the median of the number of stocks changing in each 10-stock portfolio over a sample of different OOS rebalancing dates. A low turnover for a PM strategy means that the portfolio composition does not differ much over time. Specifically, for weekly rebalancing we find that the turnover median over 396 OOS rebalancing weeks is not higher than three (out of ten) for both SKR and ETR, whilst for the other PMs is around seven. A similar result does hold under the other two rebalancing frequencies.[11]

5.4 Alternative weighting schemes

We have previously applied the naive EW portfolio rule. Here, we are interested in the relative portfolio performances, under the PMs used in the previous section, but now adopting different rules to set up portfolio weights. Thus, we consider the following schemes. First, the shortsale-constrained global-minimum-variance (GMV) portfolio, i.e. $\widehat{w}_t = \arg\min w_t^{'}\widehat{\Omega}_t w_t \;\; s.t. \; w_t^{'}l = 1$ and $w_t \geq 0$, where $\widehat{\Omega}_t$ is the estimated conditional covariance matrix of order 10 and l is a vector of ones.[12] Second, the volatility timing (VT) portfolio, i.e. $\widehat{w}_{j,t} = \left(1/\widehat{\sigma}^2_{j,t}\right)/\sum_{j=1}^{10}\left(1/\widehat{\sigma}^2_{j,t}\right)$ where $\widehat{\sigma}^2_{j,t}$ is the estimated conditional variance. Third, the reward-to-risk timing (RRT) portfolio, i.e. $\widehat{w}_{j,t} = \left(\widehat{\mu}^+_{j,t}/\widehat{\sigma}^2_{j,t}\right)/\sum_{j=1}^{10}\left(\widehat{\mu}^+_{j,t}/\widehat{\sigma}^2_{j,t}\right)$ where $\widehat{\mu}^+_{j,t} = \max(\widehat{\mu}_{j,t}, 0)$ with $\widehat{\mu}_{j,t}$ denoting the estimated conditional mean. For more details about these weighting schemes, see Kirby and Ostdiek (2012).[13]

To proceed with the weighting scheme's comparison, we compute the cumulative portfolio daily return spreads for each PM strategy under the GMV, VT and RRT schemes with respect to the EW one. These spread series only for weekly rebalancing frequency are exhibited in Figure 5.[14] Our results show that (i) the RRT scheme overall dominates the rest of the weighting schemes for all PMs consistently across the OOS period; (ii) the GMV tends to significantly underperform the other schemes for all PMs except for the ETR (80,20); (iii) the VT scheme yields portfolio returns between those obtained under the previous two weighting methods; and (iv) GMV performs less well (negative spread) than the EW portfolio for most PMs. As a result, we show that portfolio performance is significantly sensitive to alternative schemes to the naive diversification. Besides, we find similar results for the SR portfolios which are not displayed in Figure 5.

the reason why we have left out the AR component in this section.

[10] The SR portfolio, which is not included in Figure 4, shows a positive spread before the middle of the OOS period under the monthly frequency.

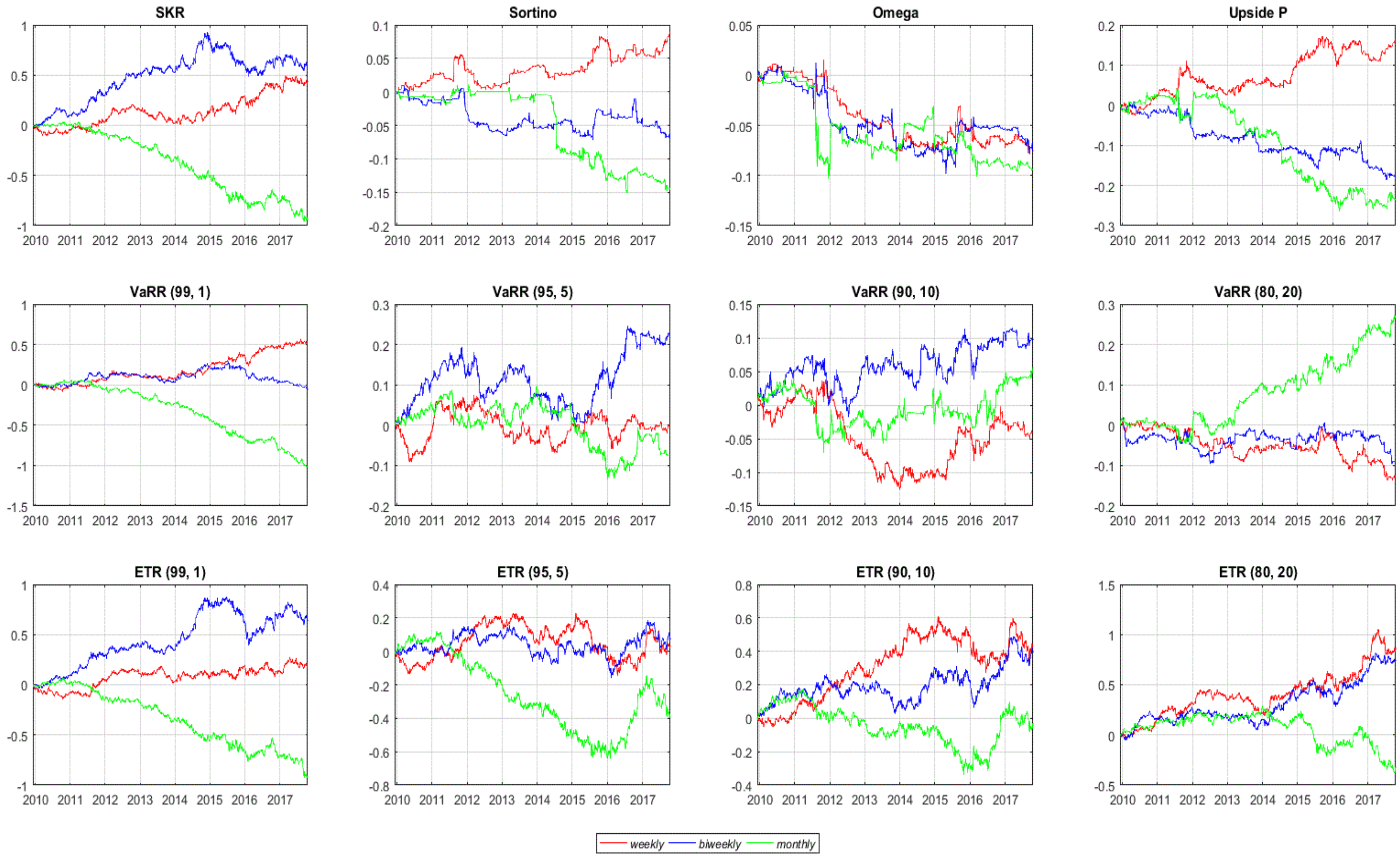
[11] Other works consider portfolios composed by the same set of stocks albeit with time-varying weights. However, the turnover here allows for different stocks to enter the portfolios, so displacing others, each rebalancing date.

[12] It is worth mentioning Carroll, Conlon, Cotter and Salvador (2017) for shortsale-constrained GMV portfolios under alternative dynamic conditional correlation (DCC) settings. They show that allocation strategies based on DCC provide performance benefits relative to EW portfolios. This interesting analysis is beyond the scope of our paper.

[13] These schemes contribute somehow to managing portfolio diversification since the classical mean variance theory (MVT) may yield portfolios that are highly concentrated. An interesting avenue for further research is obtaining *diversified* MVT portfolios by using the classical objective function of the portfolio variance augmented with a term called the diversification ratio, which is a function of weights. For more details, see Schmidt (2018).

[14] To save space, the results for biweekly and monthly rebalancing are not presented but available from the authors.

Figure 3: PM portfolio return spread with respect to SR

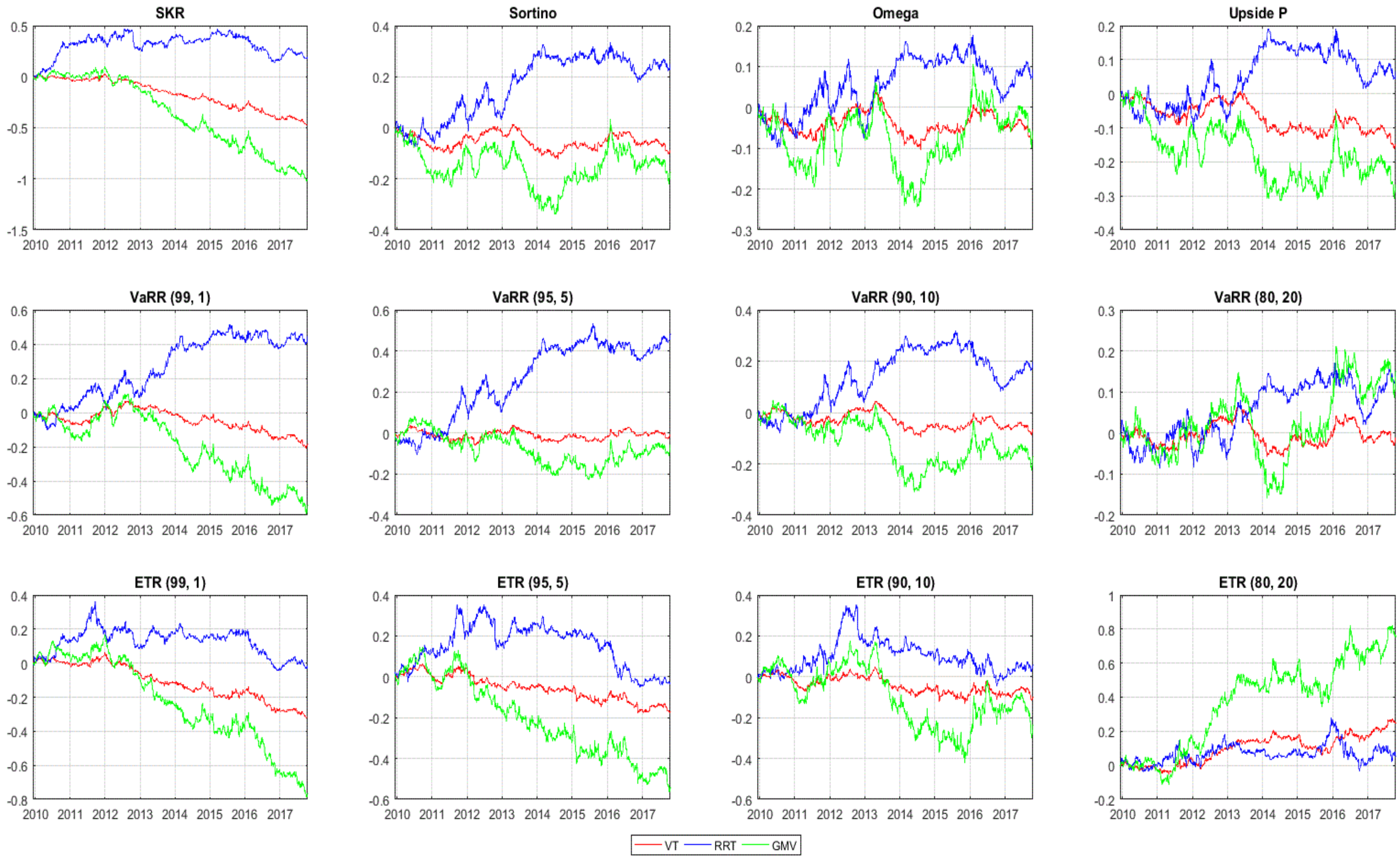


Plots of PM portfolio return spreads with respect to SR. Rebalancing frequency is weekly, biweekly and monthly over the out-of-sample period to form equally weighted portfolios of best ten stocks. Model: TV1-SNP-GJRA.

Figure 4: PM portfolio return spread with respect to S&P 100

Plots of PM portfolio return spreads with respect to S&P 100. Rebalancing frequency is weekly, biweekly and monthly over the out-of-sample period to form equally weighted portfolios of best ten stocks. Model: TV1-SNP-GJRA.

Figure 5: VT, RRT, GMV portfolio return spreads with respect to the EW portfolio



Plots of portfolio return spreads of GMV, RRT and VT weighting schemes with respect to the naive EW portfolio for each PM. We use TV1-SNP-GJRA model under weekly rebalancing.

Next, we analyze the behaviour of the four weighting schemes under the weekly rebalancing by comparing each PM strategy to the SR one. To do so, we obtain the daily conditional correlations between the portfolio return according to the selected PM and the SR portfolio return for each scheme. We apply the conditional Gaussian copula, see Patton (2006), where the marginal distributions follow the C-SNP-GJR model. In short, the copula dependency parameter (or conditional correlation in this particular case), ρ_t, is driven by an ARMA(1,q)-type process:

$$\rho_t = \Lambda\left(\gamma_0 + \gamma_1\rho_{t-1} + \gamma_2\frac{1}{q}\sum_{j=1}^{q}\Phi^{-1}\left(u_{1,t-j}\right)\Phi^{-1}\left(u_{2,t-j}\right)\right), \tag{44}$$

where $\Lambda\left(x\right) = \left(1 - e^{-x}\right)\left(1 + e^{-x}\right)^{-1}$ is the logistic transformation that keeps ρ_t within $(-1, 1)$, and $u_{i,t} = F_{i,t}\left(r_{i,t}\left|I_{t-1}\right.\right)$ $i = 1, 2$ where $F_{i,t}\left(\cdot\left|I_{t-1}\right.\right)$ denotes the conditional marginal distribution. In our study, we set $q = 8$ which is a common value adopted in previous studies, see e.g. Reboredo (2011).

As an example, Figure 6 exhibits the time series of (44) for the different PMs just under the RRT scheme. Note that the daily conditional correlations are very high for Sortino, Omega, Upside Potential and most PMs based on VaRR. Finally, those portfolios based on ETR and SKR exhibit remarkably low correlations respecting the SR portfolio, which enhance the difference between the latter and the former PMs.[15] Notably, this pattern is also observed for the other weighting schemes considered. Because of these findings, in the following section we explore the behavior of the upper/lower tail of the bivariate distribution of SR and every other PM portfolio so as to highlight possible differences in simultaneous occurrence of large/small PM portfolio returns.

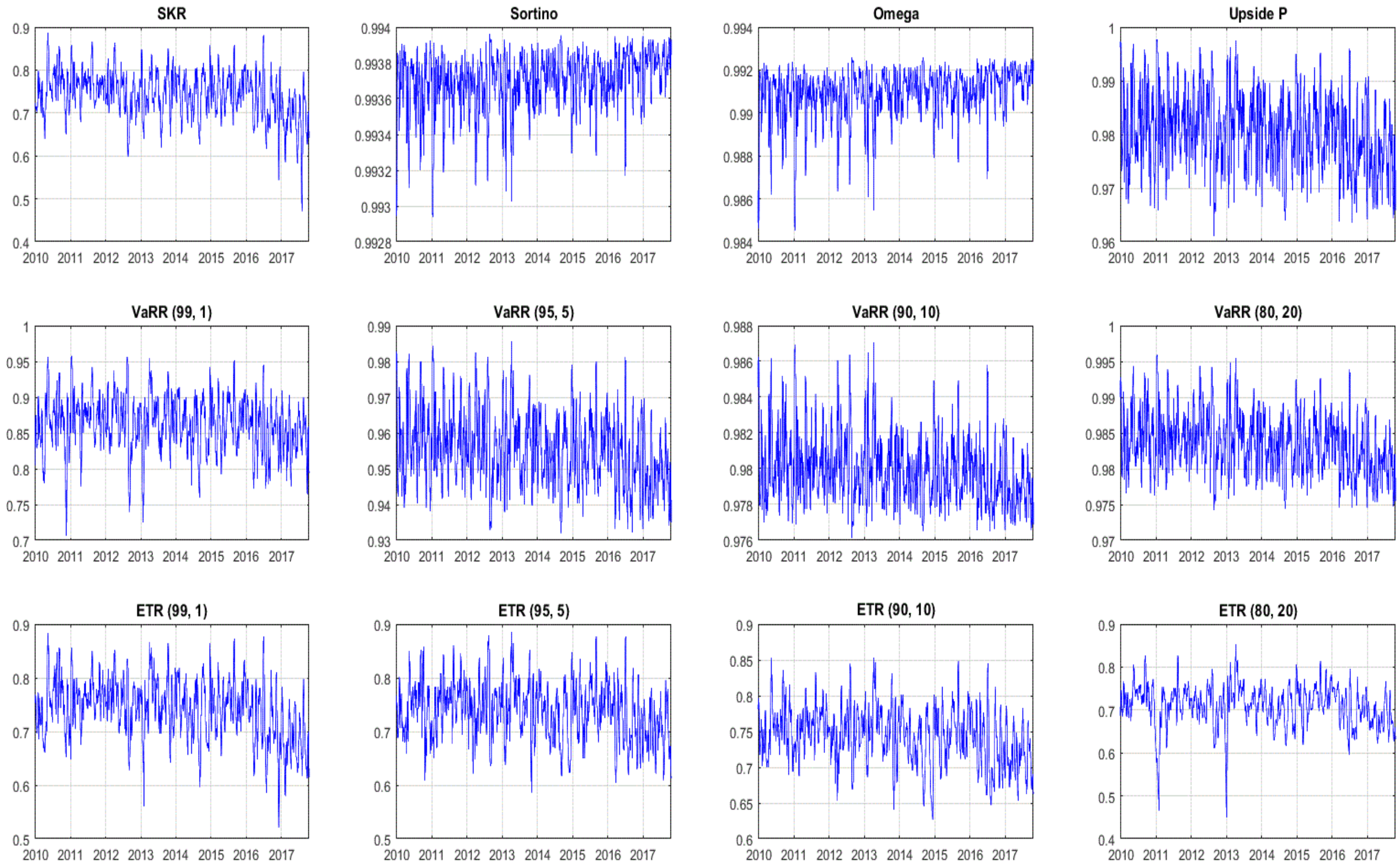
5.5 Tail dependence analysis

Next, we focus on the tail dependence measuring the probability that two variables are either in the lower or in the upper joint tails. Specifically, we study the propensity of two portfolio returns, from a given PM and SR strategies, to upward or downward comovements. This behavior is explained through the upper and lower tail dependence parameters denoted by $\lambda_U \in [0, 1]$ and $\lambda_L \in [0, 1]$, respectively. Larger values of λ_U (λ_L) indicate greater trend of the portfolio returns to cluster in the upper (lower) tail of a bivariate distribution. In such a case, the returns are said to be upper (lower) tail dependent. More precisely, λ_U (λ_L) measures the probability that a random variable –defined as a PM portfolio return– is above (below) a high (low) quantile, given that a second random variable –defined as the SR portfolio return– is above (below) a high (low) quantile. This dependence structure is modelled through copula functions.

Note that the Gaussian copula does neither capture upper nor lower dependence where the extreme tails of the distribution of the variables are independent, i.e. $\lambda_U = \lambda_L = 0$. Thus, we implement alternative copula models allowing for both/either upper or lower tail dependence. Namely, among the wide range of copula functions, we use the symmetrized Joe-Clayton (SJC), Gumbel and Clayton copulas. The SJC has both upper and lower tail dependence parameters, whilst Gumbel (Clayton) gathers only upper (lower) tail dependence. The SJC is defined directly in terms of the above probabilities. Nonetheless, both Gumbel and Clayton copulas are defined in terms of the parameters $\gamma_G > 0$ and $\gamma_C > 1$, respectively. Hence, the corresponding probabilities are given by $\lambda_U = 2 - 2^{(1/\gamma_G)}$, $\lambda_L = 0$ for the Gumbel copula and, $\lambda_U = 0$, $\lambda_L = 2^{-(1/\gamma_C)}$ for the Clayton copula, see Patton (2006, 2013).

[15] These results are also in line with those by León et al. (2018).

Figure 6: Conditional Gaussian copula correlation between PM and SR



Plots of daily correlation between PM and SR from conditional Gaussian copula. Portfolio weights obtained at weekly rebalancing through reward-to-risk timing rule.

Table 7 reports the probability estimates of the previous time-invariant copula models under the RRT scheme with weekly rebalancing.[16] We obtain the following conclusions. Firstly, for the SJC copula it is found a statistically significant and higher asymmetry value on the lower than on the upper tail, mainly for both SKR and ETR. Note that the estimates of λ_L double those of λ_U for the latter two strategies. Secondly, for Sortino, Omega, Upside potential and most VaRR cases both SJC probability coefficients are similar in magnitude as well as higher than the SKR and ETR counterparts. This means that the former PMs exhibit higher upper tail dependence respecting the SR than the latter. Thirdly, according to both Clayton and Gumbel copulas, it can be shown that both SKR and ETR exhibit statistically significant and lower values for both λ_L and λ_U than the other PMs. This evidence is in accordance with the previous results under SJC. Summing up, these findings support the superior performance (i.e., positive spread for cumulative returns) of both SKR and ETR displayed in Table 3 under the weekly rebalancing.

Table 7: Estimates for copula models (PM-SR)

PM	λ_U (SJC)	λ_L (SJC)	λ_L (Clayton)	λ_U (Gumbel)
SKR	0.38*	0.65*	0.69*	0.59*
Sortino	0.72	0.72	0.98*	0.97*
Omega	0.71	0.78	0.98*	0.93*
Upside P	0.73*	0.74*	0.95*	0.93*
VaRR (99,1)	0.58*	0.76*	0.81*	0.74*
VaRR (95,5)	0.74	0.78	0.92*	0.88*
VaRR (90,10)	0.74*	0.77	0.95*	0.93*
VaRR (80,20)	0.73*	0.78*	0.96*	0.94*
ETR (99,1)	0.37*	0.65*	0.69*	0.59*
ETR (95,5)	0.39*	0.65*	0.68*	0.59*
ETR (90,10)	0.38*	0.67*	0.70*	0.59*
ETR (80,20)	0.34*	0.61*	0.64*	0.55*

This table presents probability estimates of the parameters λ_U and λ_L (upper and lower tail dependence, respectively) for the time-invariant SJC, Gumbel and Clayton copula models (PM-SR under RRT scheme and weekly rebalancing). An asterisk (*) indicates significance at the 5% level for the implied parameters (γ_G for Gumbel, γ_C for Clayton and both λ_U and λ_L for SJC).

In order to reinforce the previous results, we estimate the time-varying SJC copula for the different PMs with respect the SR portfolio under weekly rebalancing with the RRT scheme. Following Patton (2006), the dynamics of both λ_L and λ_U under the conditional SJC copula are specified as

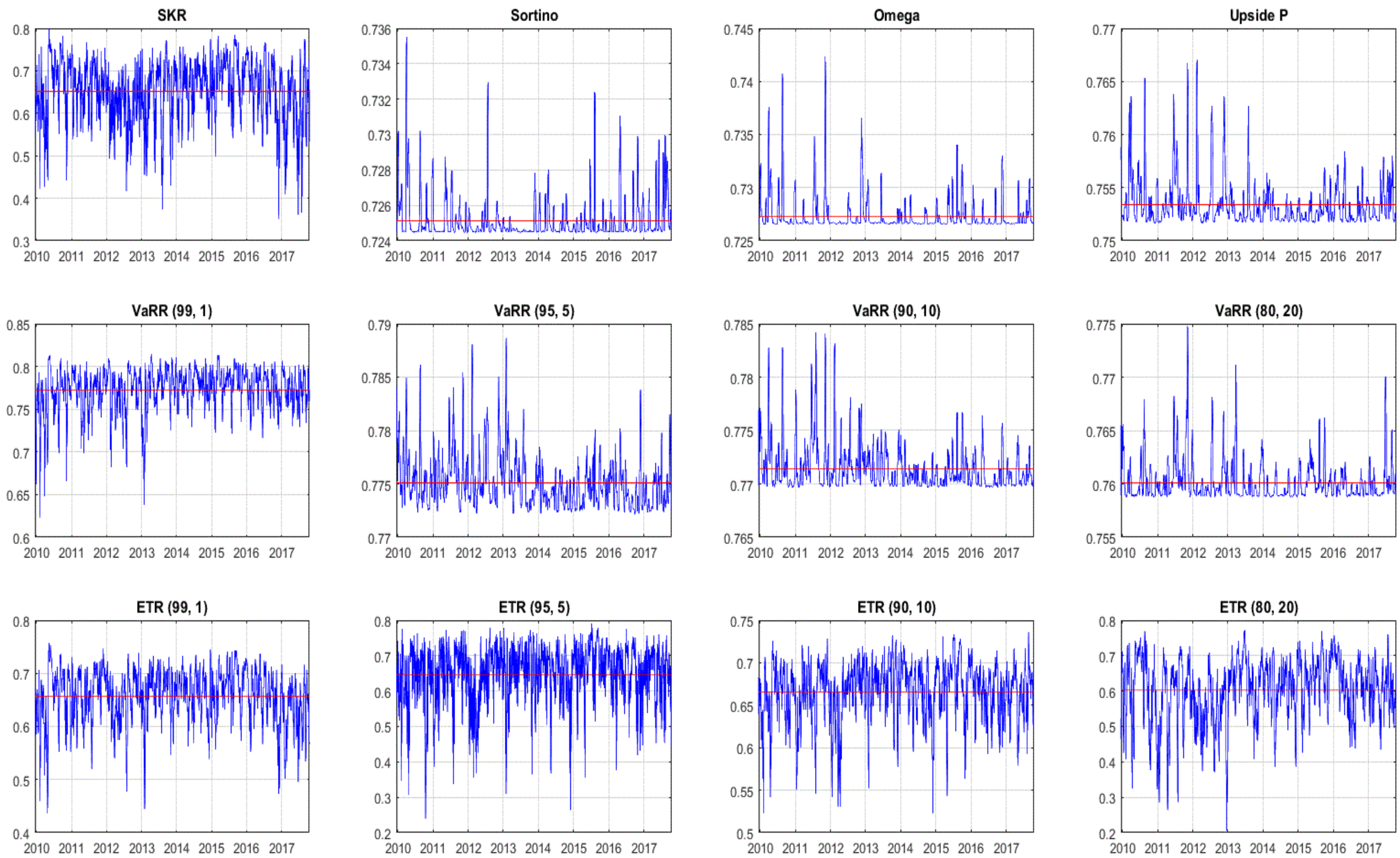
$$\lambda_{L,t} = \Delta\left(\omega_L + \beta_L \lambda_{L,t-1} + \alpha_L \frac{1}{q} \sum_{j=1}^{q} |u_{1,t-j} - u_{2,t-j}|\right), \tag{45}$$

$$\lambda_{U,t} = \Delta\left(\omega_U + \beta_U \lambda_{U,t-1} + \alpha_U \frac{1}{q} \sum_{j=1}^{q} |u_{1,t-j} - u_{2,t-j}|\right), \tag{46}$$

where $\Delta(x) = (1 + e^{-x})^{-1}$ is the logistic transformation that keeps $\lambda_{L,t}$ and $\lambda_{U,t}$ within $(0, 1)$. According to the Akaike information criterion (AIC) –not exhibited here–, the time-varying SJC estimations (see Figures 7 and 8) provide better fit than their corresponding time-invariant versions (see Table 7), except for Omega, VaRR (95,5) and VaRR (80,20) portfolios. Note that the averages of plot series in Figures 7 and 8 (red and blue horizontal lines, respectively) are rather close to the unconditional SJC estimates of λ_L and λ_U in Table 7. This new evidence corroborates the results previously found under time-invariant SJC modeling.

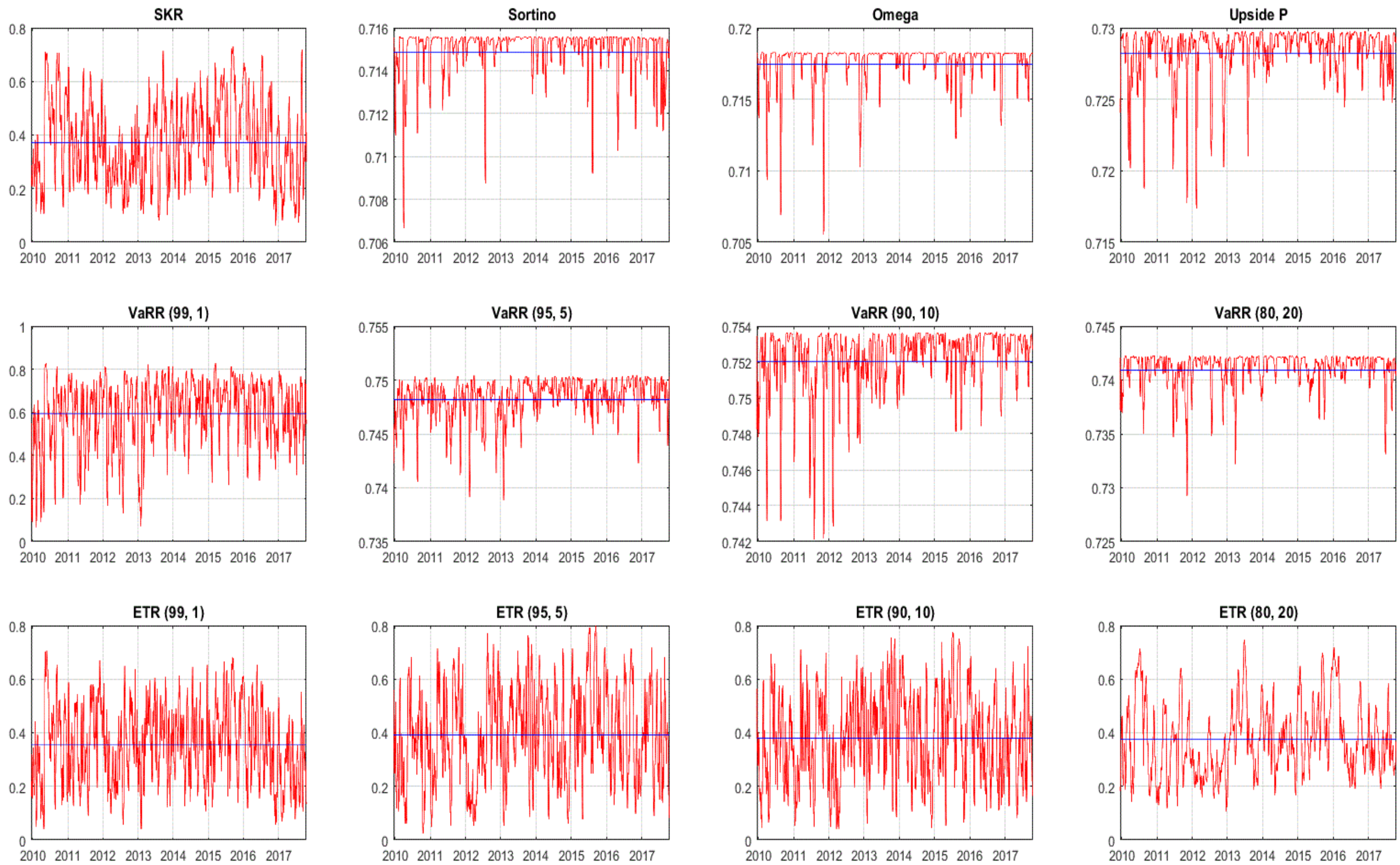
[16] Regarding the other weighting schemes considered (i.e., GMV, VT and EW), similar findings are obtained. Besides, we have estimated the Student t copula. All these results are not exhibited to save space.

Figure 7: SJC time-varying **lower** tail dependence



Plots of time-varying lower tail dependence, $\lambda_{L,t}$, for PM and SR from SJC copula. Portfolio weights obtained at weekly rebalancing through reward-to-risk timing rule. The red line represents the sample mean of the $\lambda_{L,t}$ time series.

Figure 8: SJC time-varying **upper** tail dependence



Plots of time-varying upper tail dependence, $\lambda_{U,t}$, for PM and SR from SJC copula. Portfolio weights obtained at weekly rebalancing through reward-to-risk timing rule. The blue line represents the sample mean of the $\lambda_{U,t}$ time series.

5.6 Comparative analysis

In this section we provide a comparative analysis of our TV-SNP model with reference to the historical simulation (HS) approach. To do so, we repeat the exercise presented in Section 5.3 now using HS to obtain PM portfolio return spreads as to the SR. In Figure 9 we present the results for weekly rebalancing and the EW scheme considering both constant-size rolling as well as expanding window methods. We find that the TV-SNP weekly portfolio return series in Figure 3 –represented by red lines– tend to dominate the corresponding HS ones displayed in Figure 9 consistently over the OOS period. This finding provides evidence on the relative performance of our parametric model in regard to the HS method. This is in line with some related results by Kuester, Mittnik and Paolella (2006) for VaR forecasting. Note that the expanding window HS spread outperforms the constant window one and this spread tends to be mostly positive for some PMs such as SKR, Sortino and ETRs.

6 Conclusions

This article proposes time-varying PMs obtained under an extension of the SNP pdf in León et al. (2009). Our TV-SNP-GJR model allows to account for clustering and asymmetric responses in volatility, skewness and kurtosis. We analyze its statistical properties and provide closed-form expressions of conditional partial moments, quantiles and expected shortfall. Through an application to stock-index and foreign-exchange returns, we show that higher-order moments' clustering and asymmetric response to positive and negative shocks are significant features. We corroborate these results through an original skewness and kurtosis NIC analysis.

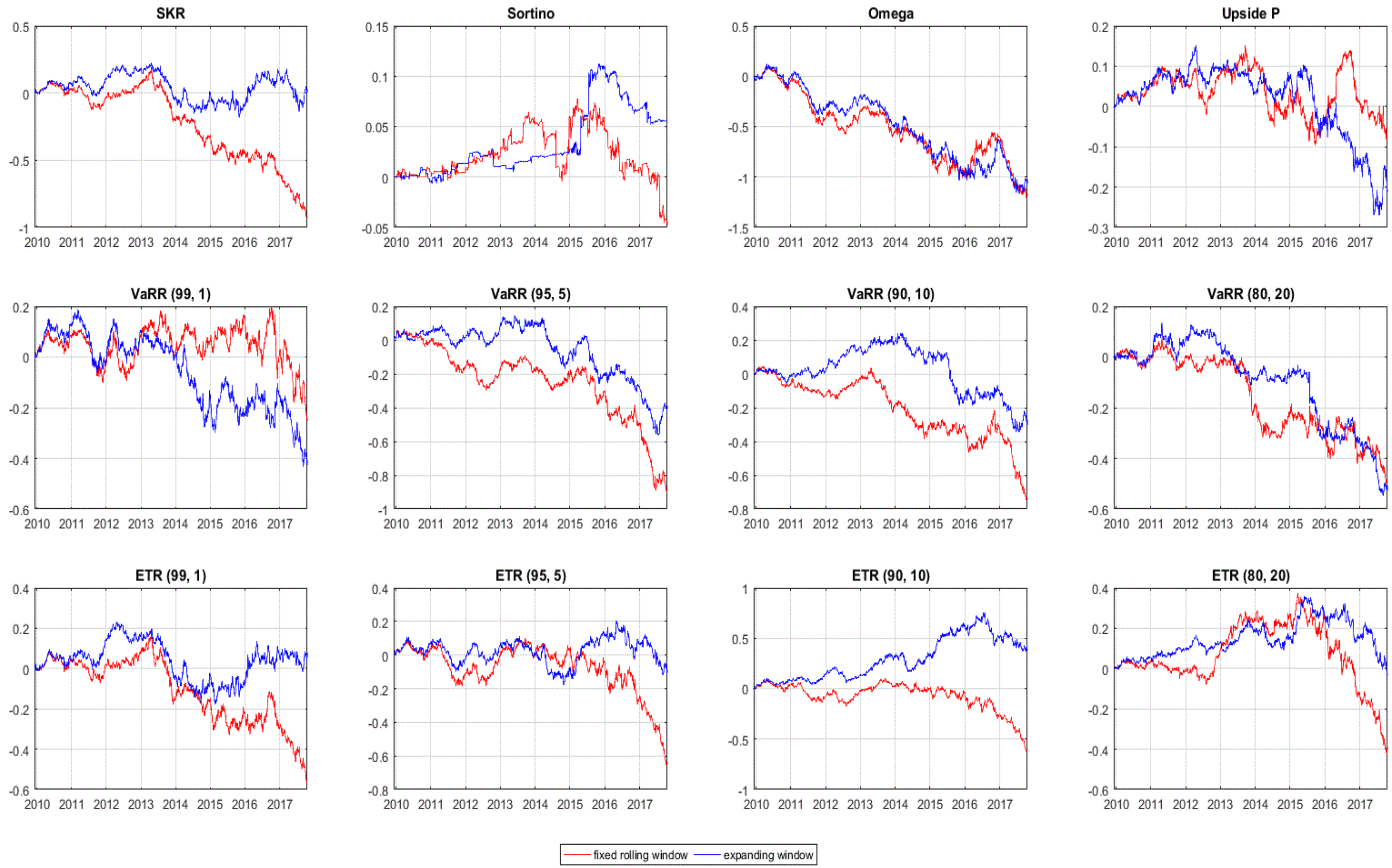
The performance of our model is tested through an out-of-sample application to portfolios formed from ranking stocks from the S&P 100 index. Our results reveal that the asset allocation critically depends on the PM considered as well as on the portfolio rebalancing period. We show that not all PMs yield greater portfolio cumulative returns. These PMs tend to improve less the portfolios based on the SR strategy the lower the rebalancing frequency. Importantly, for monthly frequency SKR and ETR do not improve SR strategies, whilst for weekly rebalancing they yield portfolio selections that significantly beat SR portfolios. We also show that portfolio performance is significantly sensitive to alternative weighting schemes. Finally, a comparative analysis respecting the approach of historical simulation highlights the relative performance of our parametric model.

There are at least three interesting avenues for further research. The first involves a SNP analysis of transaction costs and estimation error on portfolio screening according to the approaches proposed by DeMiguel, Garlappi and Uppal (2009) and Olivares-Nadal and DeMiguel (2018). The second is about the portfolio diversification following the methodologies proposed by Kolm, Tütüncü and Fabozzi (2014) and Schmidt (2018). The third is the portfolio evaluation under alternative PMs and weighting schemes based on the tail risk exposure –CoVaR by Adrian and Brunnermeier (2016) and the marginal ES measure by Acharya, Pedersen, Philippon and Richardson (2017) –as in Hwang, Xu and In (2018).

Acknowledgements

We are grateful to participants at the XXVII Annual Symposium of the Society of Nonlinear Dynamics and Econometrics, University of Alicante Workshop in Quantitative Economics and 36th International Symposium on Forecasting for helpful comments, discussions and suggestions. Of course, the usual caveat applies. Financial support from the Spanish Ministry of Economy and Competitiveness through grants ECO2017-87069-P and ECO2016-75631-P is gratefully acknowledged by the first and second authors, respectively.

Figure 9: PM portfolio return spread with respect to SR (Historical simulation)



Plots of PM portfolio return spreads with respect to SR. Rebalancing frequency is weekly over the out-of-sample period to form EW portfolios of best ten stocks. The fixed-rolling window is the same as that used to obtain the portfolio returns depicted in Figure 3. Model: HS.

Appendix

i) Obtain the expression of $m_k(\cdot)$:

Let $x \sim N(0,1)$ with $\phi(\cdot)$ and $\Phi(\cdot)$ as pdf and cdf, respectively. We are interested in the moments of the truncated Normal random variable defined as $x \mid x \leq u$ where $u \in \mathbb{R}$. Thus, $m_k(u) \equiv E_\phi\left[x^k \left| x \leq u\right.\right]$ where $k \in \mathbb{N}$. A recursive formula for the truncated normal moments can be obtained as

$$m_k(u) = (k-1)\, m_{k-2}(u) - \frac{u^{k-1}\phi(u)}{\Phi(u)}, \qquad k = 1,2,3,... \tag{47}$$

where $m_{-1}(u) = 0$ and $m_0(u) = 1$. For more details, see Liquet and Nazarathy (2015).

ii) Obtain the expression of $\xi_j(\cdot)$:

Let $\xi_j(u) = \int_{-\infty}^{u} x^j q(x)\, dx$ where $j \in \mathbb{N}$ and $q(\cdot)$ is the pdf in (5), then

$$\begin{aligned}
\xi_j(u) &= \int_{-\infty}^{u} x^j q(x)\, dx \\
&= \sum_{k=0}^{4} \gamma_k \int_{-\infty}^{u} x^j H_k(x)\, \phi(x)\, dx \\
&= \Phi(u) \sum_{i=1}^{5} \eta_i m_{j+i-1}(u),
\end{aligned} \tag{48}$$

such that $m_k(u)$ is defined in (47) and

$$\begin{aligned}
\eta_1 &= 1 - \frac{\gamma_2}{\sqrt{2}} + \frac{3\gamma_4}{\sqrt{4!}}, \\
\eta_2 &= \gamma_1 - \frac{3\gamma_3}{\sqrt{3!}}, \\
\eta_3 &= \frac{\gamma_2}{\sqrt{2}} - \frac{6\gamma_4}{\sqrt{4!}}, \\
\eta_4 &= \frac{\gamma_3}{\sqrt{3!}}, \quad \eta_5 = \frac{\gamma_4}{\sqrt{4!}},
\end{aligned} \tag{49}$$

where γ_k can be seen in (7). Note that $\xi_0(u) = \Phi(u) \sum_{i=1}^{5} \eta_i m_{i-1}(u)$ is just the SNP cdf given in (11).

iii) Proof of Proposition 3: The expected shortfall, $ES_t(\alpha)$, is obtained as

$$\begin{aligned}
E_{t-1}\left(r_t \left| r_t \leq r_{\alpha,t}\right.\right) &= \frac{1}{\alpha} \int_{-\infty}^{r_{\alpha,t}} r_t f\left(r_t \left| I_{t-1}; \psi\right.\right) dr_t \\
&= \frac{1}{\alpha} \int_{-\infty}^{r_{\alpha,t}^*} \left(\mu_t + a_t\sigma_t + b_t\sigma_t x_t\right) q\left(x_t \left| I_{t-1}\right.\right) dx_t \\
&= \kappa_{0t} + \frac{\kappa_{1t}}{\alpha} \int_{-\infty}^{r_{\alpha,t}^*} x_t q\left(x_t \left| I_{t-1}\right.\right) dx_t \\
&= \kappa_{0t} + \frac{\kappa_{1t}}{\alpha} \xi_{1t}\left(r_{\alpha,t}^*\right) && (50) \\
&= \kappa_{0t} + \frac{\kappa_{1t}}{\alpha} \Phi\left(r_{\alpha,t}^*\right) \sum_{i=1}^{5} \eta_{it} m_{1+i-1}\left(r_{\alpha,t}^*\right) && (51)
\end{aligned}$$

where $r_{\alpha,t}^* = \left(r_{\alpha,t} - \kappa_{0t}\right)/\kappa_{1t}$, $\kappa_{0t} = \mu_t + a_t\sigma_t$, $\kappa_{1t} = b_t\sigma_t$ and $\xi_{1t}(u)$ in (50) is computed according to $\xi_1(u)$ in (48) such that η_{it} in (51) is given by the expression of η_i in (49) but replacing v_i with $v_{i,t}$, and finally, $m_j(\cdot)$ in (51) is defined in (47).

References

[1] Acharya, V., Pedersen, L., Philippon, T., Richardson, M., 2017. Measuring systemic risk. *Review of Financial Studies* 30, 2-47.

[2] Adrian, T., Brunnermeier, M.K., 2016. CoVaR. *American Economic Review*, 106 (7), 1705-1741.

[3] Albuquerque, R., 2012. Skewness in stock returns: Reconciling the evidence on firm versus aggregate returns. *Review of Financial Studies* 55 (5), 1630-1673.

[4] Anatolyev, S., Petukhov, A., 2016. Uncovering the skewness news impact curve. *Journal of Financial Econometrics* 14 (4), 746-771.

[5] Auer, B.R., 2015. On the role of skewness, kurtosis, and the location and scale condition in a Sharpe ratio performance evaluation setting. *International Journal of Theoretical and Applied Finance* 18 (6), 1-13.

[6] Bacon, C.R., 2008. *Practical portfolio performance measurement and attribution*, second edition. Wiley, New York.

[7] Biglova, A., Ortobelli, S., Rachev, S., Stoyanov, S., 2004. Different approaches to risk estimation in portfolio theory. *Journal of Portfolio Management* 31 (1), 103-112.

[8] Bruni, R., Cesarone, F., Scozzari, A., Tardella, F., 2017. On exact and approximate stochastic dominance strategies for portfolio selection. *European Journal of Operational Research* 259 (1), 322-329.

[9] Caporin, M., Jannin, G.M., Lisi, F., Maillet, B.B., 2014. A survey on the four families of performance measures. *Journal of Economic Surveys* 28 (5), 917-942.

[10] Caporin, M., Lisi, F., 2011. Comparing and selecting performance measures using rank correlations. *The Open-Access, Open-Assessment E-Journal* 5, 1-34.

[11] Carnero, M.A., Peña, D., Ruiz, E., 2004. Persistence and kurtosis in GARCH and stochastic volatility models. *Journal of Financial Econometrics* 2 (2), 319-342.

[12] Carroll, R., Conlon, T., Cotter, J., Salvador, E., 2017. Asset Allocation with correlation: A composite trade-off. *European Journal of Operational Research* 262, 1164-1180.

[13] DeMiguel, V., Garlappi, L., Uppal, R., 2009. Optimal versus naive diversification: How inefficient is the $1/N$ portfolio strategy. *Review of Financial Studies* 22, 1915-1953.

[14] Eling, M., 2008. Does the measure matter in the mutual fund industry? *Financial Analysts Journal* 64 (3), 54-66.

[15] Eling, M., Schuhmacher, F., 2007. Does the choice of performance measure influence the evaluation of hedge funds? *Journal of Banking and Finance* 31, 2632-2647.

[16] Farinelli, S., Tibiletti, L., 2008. Sharpe thinking in asset ranking with one-sided measures. *European Journal of Operational Research* 185, 1542-1547.

[17] Feunou, B., Jahan-Parvar, M.R., Tédongap, R., 2016. Which parametric model for conditional skewness? *European Journal of Finance* 22 (13), 1237-1271.

[18] Fishburn, P.C., 1977. Mean-risk analysis with risk associated with below-target returns. *American Economic Review* 67 (2), 116-126.

[19] Gallant, A.R., Nychka, D.W., 1987. Semi-nonparametric maximum likelihood estimation. *Econometrica* 55, 363-390.

[20] Glosten, R.T., Jagannathan, R., Runkle, D., 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance* 48 (5), 1779-1801.

[21] Hansen, B.E., 1994. Autoregressive conditional density estimation. *International Economic Review* 35 (3), 705-730.

[22] Harvey, C.R., Siddique, A., 1999. Autoregressive conditional skewness. *Journal of Financial and Quantitative Analysis* 34 (4), 465-487.

[23] He, C., Terasvirta, T., 1999. Properties of moments of a family of GARCH processes. *Journal of Econometrics* 92, 173-192.

[24] Hwang, I., Xu, S., In, F., 2018. Naive versus optimal diversification: Tail risk and performance. *European Journal of Operational Research* 265 (1), 372-388.

[25] Israelsen, C.L., 2005. A refinement of the Sharpe ratio and information ratio. *Journal of Asset Management* 5 (6), 423-427.

[26] Jondeau, E., Rockinger, M., 2003. Conditional volatility, skewness and kurtosis: Existence, persistence, and comovements. *Journal of Economic Dynamics and Control* 27, 1699-737.

[27] Keating, C., Shadwick, W.F., 2002. A universal performance measure. *Journal of Performance Measurement* 6 (3), 59-84.

[28] Kirby, C., Ostdiek, B., 2012. It's all in the timing: Simple active portfolio strategies that outperform naive diversification. *Journal of Financial and Quantitative Analysis* 47, 437-467.

[29] Kolm, P.N., Tütüncü, R., Fabozzi, F.J., 2014. 60 Years of portfolio optimization: Practical challenges and current trends. *European Journal of Operational Research* 234, 356-371.

[30] Kuester, K., Mittnik, S., Paolella, M.S., 2006. Value-at-risk prediction: A comparison of alternative strategies. *Journal of Financial Econometrics* 4 (1), 53-89.

[31] Lalancette, S., Simonato, J.G., 2017. The role of the conditional skewness and kurtosis in VIX index valuation. *European Financial Management* 23 (2), 325-354.

[32] León, A., Rubio, G., Serna, G., 2005. Autoregressive conditional volatility, skewness and kurtosis. *Quarterly Review of Economics and Finance* 45, 599-618.

[33] León, A., Mencía, J., Sentana, E., 2009. Parametric properties of semi-nonparametric distribution, with applications to option valuation. *Journal of Business & Economic Statistics* 27 (2), 176-192.

[34] León, A., Moreno, M., 2017. One-sided performance measures under Gram-Charlier distributions. *Journal of Banking and Finance* 74, 38-50.

[35] León, A., Navarro, L., Nieto, B., 2018. Screening rules and portfolio performance. *North American Journal of Economics and Finance*, forthcoming.

[36] Liquet, B., Nazarathy, Y., 2015. A dynamic view to moment matching of truncated distributions. *Statistics and Probability Letters* 104, 87-93.

[37] Oh, D.H., Patton, A.J., 2017. Modeling dependence in high dimensions with factor copulas. *Journal of Business & Economic Statistics* 35 (1), 139-154.

[38] Olivares-Nadal, A.V., DeMiguel, V., 2018. A robust perspective on transaction costs in portfolio optimization. *Operations Research* 66 (3), 733-736.

[39] Ortobelli, S., Vitali, V., Cassader, M., Tichy, T., 2018. Portfolio selection strategy for fixed income markets with immunization on average. *Annals of Operations Research* 260, 395-415.

[40] Patton, A., 2006. Modeling asymmetric exchange rate dependence. *International Economic Review* 47, 527-556.

[41] Patton, A., 2013. Copula methods for forecasting multivariate time series, in G. Elliott and A. Timmermann (eds.). *Handbook of Economic Forecasting* Volume 2, Springer Verlag.

[42] Pézier, J., White, A., 2008. The relative merits of alternative investments in passive portfolios. *Journal of Alternative Investments* 10 (4), 37-49.

[43] Reboredo, J.M., 2011. How do oil price co-move? A copula approach. *Energy Economics* 33, 948-955.

[44] Schmidt, A.B., 2018. Managing portfolio diversity within the mean variance theory. *Annals of Operations Research,* forthcoming.

[45] Sharpe, W.F., 1966. Mutual fund performance. *Journal of Business* 39, 119-138.

[46] Sharpe, W.F., 1994. The Sharpe ratio. *Journal of Portfolio Management* 21, 49-58.

[47] Sortino, F.A., van der Meer, R., 1991. Downside risk. *Journal of Portfolio Management* 17 (4), 27-31.

[48] Sortino, F.A., van der Meer, R., Platinga, A., 1999. The Dutch triangle. *Journal of Portfolio Management* 26, 50-57.

[49] Theodossiou, P., 1998. Financial data and the skewed generalised T distribution. *Management Science* 44, 1650-1661.

[50] Theodossiou, P., Savva, C.S., 2016. Skewness and the relation between risk and return. *Management Science* 62 (6), 1598-1606.

[51] Watanabe, Y., 2006. Is Sharpe ratio still effective? *Journal of Portfolio Measurement* 11 (1), 55-66.

[52] Zakamouline, V., 2011. The performance measure you choose influences the evaluation of hedge funds. *Journal of Performance Measurement* 15 (3), 48-64.

[53] Zakoïan, J.M., 1994. Threshold heteroskedastic models. *Journal of Economic Dynamics and Control* 18 (5), 931-955.